L6 - Solving Exp Equations with Logs

January-14-16
7:53 AM

Unit 7: Exponents & Logarithms
Lesson 6 Solving Exponential Equations Using Logarithms

If we are solving an exponential equation that cannot match the base, we can now use logarithms (finally!)

Ex.1: Solve for x

a) $7^{x+1} = 9$

$\rightarrow \log_7 9 = x+1$

$x = \log_7 9 - 1 \approx 0.129$

(TI-83) $= \dfrac{\log 9}{\log 7} - 1$

b) $3^{x+2} = 5^{2x-3}$ * log both sides

$\log 3^{x+2} = \log 5^{2x-3}$

$(x+2)\log 3 = (2x-3)\log 5$

$x\log 3 + 2\log 3 = 2x\log 5 - 3\log 5$

$2\log 3 + 3\log 5 = 2x\log 5 - x\log 3$ * factor

$2\log 3 + 3\log 5 = x(2\log 5 - \log 3)$

$x = \dfrac{2\log 3 + 3\log 5}{2\log 5 - \log 3} \approx 3.31$

c) $3(8^{x-2}) = 5^{1+x}$

$\log 3(8^{x-2}) = \log 5^{1+x}$

$\log 3 + \log 8^{x-2} = \log 5^{1+x}$

$\log 3 + (x-2)\log 8 = (1+x)\log 5$

$\log 3 + x\log 8 - 2\log 8 = \log 5 + x\log 5$

$x\log 8 - x\log 5 = \log 5 + 2\log 8 - \log 3$

$\rightarrow x(\log 8 - \log 5) = \log 5 + 2\log 8 - \log 3$

$x = \dfrac{\log 5 + 2\log 8 - \log 3}{\log 8 - \log 5} \approx 9.94$

There are many applications of logarithms as well… just as with exponential functions.

Ex. 2: Strontium-90 has a half-life of 28 years. How long will it take a 85 g sample to decay to 15g?

$P(t) = P_0(0.5)^{t/n}$

$n = 28$

$P_0 = 85$

$P(t) = 15$

$\dfrac{15}{85} = \dfrac{85}{85}(0.5)^{t/28}$

$\dfrac{3}{17} = (0.5)^{t/28}$

$\rightarrow \log_{0.5}\dfrac{3}{17} = \dfrac{t}{28}$

$t = 28\log_{0.5}\dfrac{3}{17}$

≈ 70.1 years

Ex. 3: How long would it take a population of bees to triple if they multiply 8 fold every 5 weeks?

$P(t) = P_0(r)^{t/n}$

$r = 8$

$n = 5$

$P(t) = 3P_0$ (triple)

$3P_0 = P_0(8)^{t/5}$

$3 = (8)^{t/5}$

$\rightarrow \log_8 3 = \frac{t}{5}$

$t = 5\log_8 3$

≈ 2.64 weeks

Ex. 4: How much more intense is an earthquake of 9.8 than an earthquake of 5.4 on the Richter Scale?

provided

$\text{Intensity} = \frac{10^a}{10^b}$

a, b = Richter Scale

$I = \frac{10^{9.8}}{10^{5.4}}$

$I = 10^{4.4}$

≈ 25118.9

Ex. 5: If an earthquake registers 2.7, how high on the Richter scale does an earthquake 52 000 times stronger register at?

$52000 = \frac{10^a}{10^{2.7}}$

$52000 = 10^{a-2.7}$

$\rightarrow \log_{10} 52000 = a - 2.7$

$a = \log 52000 + 2.7$

$a = 7.42$

Practice: Pg. 136 – 137 # 1 – 21 (Omit # 6, 7, 10)