L7 - Natural Log & e

March-02-15
12:26 PM

Unit 7: Exponents & Logarithms
Lesson 7 Natural Logarithm & the Number e

e is defined as the number that $\left(1+\frac{1}{n}\right)^n$ approaches as n gets larger and larger without bound.

Like π, the number e is irrational. It is used in growth and decay situations that are continuous, such as the increase in world population or the decrease in the mass of a snowball as it melts. The number e (to nine decimals accuracy) can be found on your calculator with the $\boxed{e^x}$ button.

$e = 2.718281828...$

You will meet e again in a more meaningful way when you take calculus.
With calculus, mathematicians have developed the following formula for continuous growth or decay:

$\boxed{A = A_0 e^{kt}}$ ← provided

$(k = r)$

Where A_0 is the initial amount, k is a constant (usually a growth rate or a decay rate), and A is the new amount resulting after a certain time t.

eg. Venezuela

Ex.1: The population of a particular country is 25 million. Assuming the population is growing continuously, the population P, in millions, t years from now can be determined by the formula $P = 25e^{0.022t}$. What will be the population, in millions, 20 years from now?

$P = 25e^{0.022t}$

Initial Population

0.022 = Growth Rate

2.2%

*Not 1.022

$t = 20$

$P = 25e^{0.022(20)}$

$P = 38.82$

38.82 million people!

Natural Logarithms

Logarithms with the base of e are called natural logarithms. The natural logarithm of a number x is written as $\ln x$ (pronounced "lawn x").

In other words, $\log_e x = \ln x$.

You should be able to evaluate the following without a calculator:

$\ln e = \log_e e = 1$ $\quad \ln e^4 = 4$ $\quad \ln 1 = 0$ $\quad e^{\ln 202} = 202$

$\rightarrow \log_e ? = \ln 202$

$\ln ? = \ln 202$

Natural logs obey the same laws and properties as all other logarithms.

$$\ln x + \ln y = \ln(xy) \qquad \ln x - \ln y = \ln\left(\frac{x}{y}\right) \qquad \ln x^n = n \ln x \qquad e^{\ln x} = x$$

Ex.2: Solve for x to 3 decimal places

a) $3 = e^{7x}$

$\rightarrow \log_e 3 = 7x$

$\boxed{x = \frac{\ln 3}{7}} = 0.157$

b) $\frac{1500}{5} = \frac{\cancel{5}e^{0.045x}}{\cancel{5}}$

$300 = e^{0.045x}$

$\log_e 300 = 0.045x$

$\boxed{x = \frac{\ln 300}{0.045}} = 126.751$

c) $\frac{\ln 3.6}{0.034} = \frac{\cancel{0.034}x}{\cancel{0.034}}$

$\boxed{x = \frac{\ln 3.6}{0.034}} = 37.675$

Ex.3: In 1997,the population of Calgary was 795 000. This was an increase of 3.06% from the previous year. Assuming that the population is growing continuously, the population P, in thousands, can be determined by the formula $P = 795e^{0.0306t}$ where t is the time in years since 1997.

a) What is the population of Calgary in the year 2000?

$t = 3$

(2000-1997)

$P = 795e^{0.0306(3)}$

$P = 871.44$ (thousand)

871440 people.

b) What year would the population exceed one million?

$P = 1000\,000$

$= 1000$ (thousands)

$\frac{1000}{795} = \frac{\cancel{795}e^{0.0306t}}{\cancel{795}}$

$\frac{200}{159} = e^{0.0306t}$

$\rightarrow \log_e \frac{200}{159} = 0.0306t$

$t = \frac{\ln 200/159}{0.0306} = 7.497$

1997+7.5

About June of 2004

c) If the population grew at 4.01%, how would the formula change?

$P = 795e^{0.0401t}$

Ex. 4: Express 7 as a power of e.

$7 = e^x$

$\rightarrow \log_e 7 = x$

$\ln 7 = x$

$\therefore \; 7 = e^{\ln 7}$

"therefore"

Ex. 5: Express e as a power of 12.

$e = 12^x$

$\rightarrow \log_{12} e = x$

$\therefore \; e = 12^{\log_{12} e}$

Practice: Study for QUEST! + Log Review WKST