Notes 1

October-28-15
12:24 PM

8.1: Systems of Linear Equations and Graphs

Systems of Equations

- Two or more linear equations studied together is called a **systems of equations.**
- The following is a system of equations:

$$\begin{cases} x + y = 8 & (1) \\ 3x - 2y = 14 & (2) \end{cases}$$

plug it in → true statement

- The solution to a system of equations is an ordered pair (x,y) that satisfies **BOTH** equations. To check that the ordered pair satisfies both equations substitute the values for x and y into each equation.

Check if either of the ordered pairs (3, -1) or (6, 2) satisfies the system above.

Check $(3,-1)$:
(1) $3+(-1)=8$ ✗
(2) $3(3)-2(-1)=14$ ✗
→ $(3,-1)$ is not a solution

Check $(6,2)$:
(1) $6+2=8$ ✓
(2) $3(6)-2(2)=14$ ✓
→ $(6,2)$ is a solution

3 methods to solving systems:
1. Graphing (today)
2. Substitution (9.1)
3. Addition / Subtraction (9.2: Elimination method)

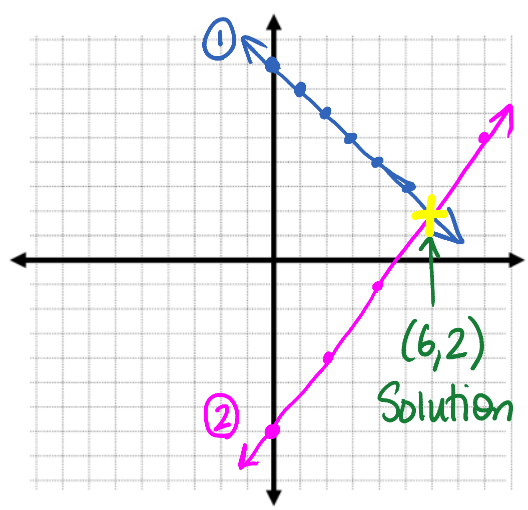
Graphing: (by hand and using the calculator)

Ex. 1-Graphing by hand:

A) Create a table of values for each equation. (OR, use y=mx+b)

(1) $\cancel{x} + y = 8$
$-\cancel{x}$ $\quad -x$
$y = -x + 8$
slope $= -1/1$
y-int $= 8$

(2) $\cancel{3x} - 2y = 14$
$-\cancel{3x}$ $\quad -3x$
$\frac{\cancel{-2}y}{\cancel{-2}} = \frac{-3x}{-2} + \frac{14}{-2}$
$y = \frac{3}{2}x - 7$
slope $= 3/2$
y-int $= -7$



B) Draw a line for each equation on the graph. Identify the intersection point of the two lines.

1

The solution to the system of equations is all of the above:

- The point of intersection of the lines on a graph.
- An ordered pair that satisfies both equations.
- A pair of values occurring in the tables of values of both equations.

Ex. 2: Solve the system with a graphing calculator.

a) **Rearrange both equations by hand so they are in the form y = mx + b**
b) **Enter the first equation into y_1= and the second equation into y_2=**
c) **Press graph**
d) **Press 2^{nd}, Trace, 5-Intersect**
e) **Press enter for left-bound, enter for right-bound, enter for 'guess' and the point of intersection will appear.**

① $3y - \cancel{6x} = -15$

$\quad \cancel{+6x} \quad +6x$

$\frac{\cancel{3}y}{\cancel{3}} = \frac{6x}{3} - \frac{15}{3}$

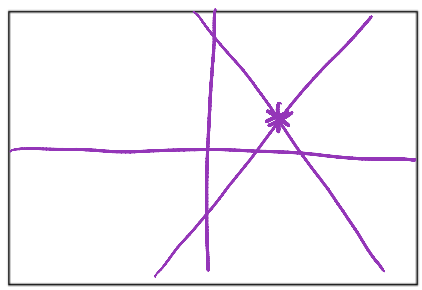
$y = 2x - 5$

② $2y + \cancel{3x} = 11$

$\quad \cancel{-3x} \quad -3x$

$\frac{\cancel{2}y}{\cancel{2}} = \frac{-3x}{2} + \frac{11}{2}$

$y = -\frac{3}{2}x + \frac{11}{2}$



[−10 , 10] [−10 , 10]
x min x max y min y max

Window dimensions

Equations entered

$Y_1 = 2x - 5$

$Y_2 = -3/2\,x + 11/2$

Solution: (3, 1)

Ex. 3: Solve the system with a graphing calculator.

① $3y + \cancel{2x} = 5$

$\quad \cancel{-2x} \quad -2x$

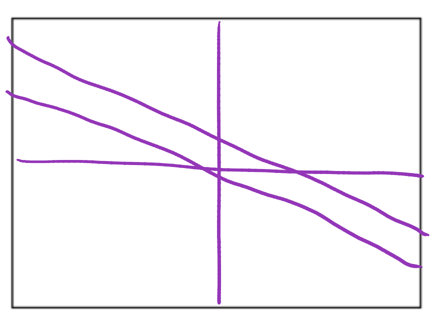
$\frac{\cancel{3}y}{\cancel{3}} = \frac{-2x}{3} + \frac{5}{3}$

$y = \frac{-2}{3}x + \frac{5}{3}$

② $\cancel{-6}y = 4x + 2$

$\quad \cancel{-6} \quad -6 \quad -6$

$y = -\frac{2}{3}x - \frac{1}{3}$



[−10 , 10] [−10 , 10]
x min x max y min y max

$Y_1 = -2/3X + 5/3$

$Y_2 = -2/3X - 1/3$

→Parallel!

→No Intersection!

→No Solution!

Ex. 4- Creating a system * Write a "let" statement

Sara has saved $25 and her friend Clint has saved $40. They have just started part-time jobs together. Each day that they work Sara adds $6 to her savings, while Clint adds $5. They want to know if they will ever have the same amount of money. If so, what will the amount be and on what day?

EQUATION 1: (Sara) $M = 25 + 6d$

EQUATION 2: (Clint) $M = 40 + 5d$

y x

let M = money in account

d = days

To solve → graph! (intersection)

d M

Intersection: (15, 115)

After 15 days, they will both have $115.

~~Ex. 5~~: **Page 431 # 19 – Discuss in groups and write your answer below.**

When would plan A be the best?

When would plan B be the best?

When would plan C be the best?

<u>Practice Questions:</u>

-Use graph paper for # 3, 6, 10

-You may use the graphing calculator for #7 and to check any answer.

-A sketch of the calculator screen, the equations you entered and the window dimensions must be included.

Pages 427 to 428 # 3, 5, 6, 7ac, 10

8.2: Modeling and Solving Linear Systems

Tips for creating a system of linear equations:

1) Assign variables that are meaningful to the context of the problem. Use let statements.
   Example: "Let V= volume"

2) Draw a diagram or make a table to organize information given in the problem. Remember you are creating TWO equations with the same variables in each equation.

3) Look for a constant and a rate of change (slope).
   In y=mx+b, m is the rate of change and b is the constant.
   (slope) (y-int)

   Example: A car rental company charges $40 per day plus $0.50 per kilometer.
   C = 40 + 0.50k where C = cost and k= # of kilometers
   ↓ ↓ ↓ ↙
   y = b + mx

4) The intersection of the two graphs is the solution.
   (use graphing calc. or by hand)

5) The solution can be checked by subbing the x and y value back into the original equations and seeing if it makes them true.

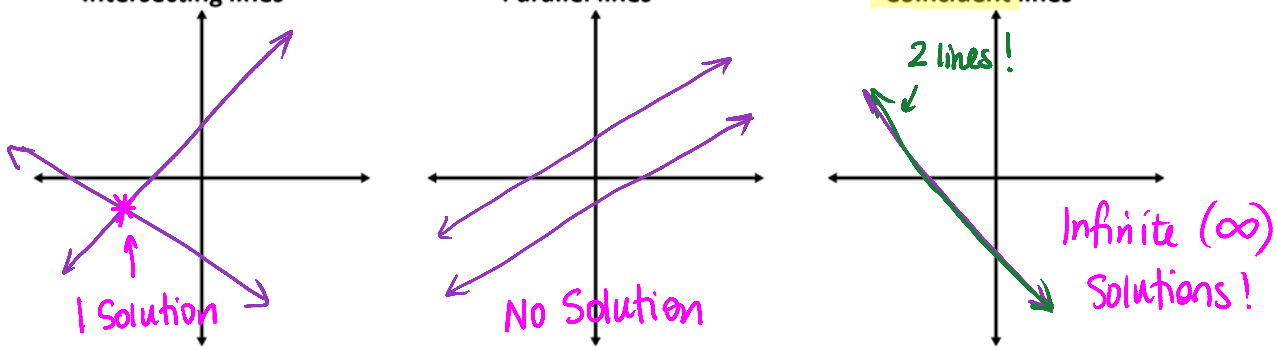
Group practice questions:
Page 440 # 1, 2b, 3, 4a

Individual practice questions:
Page 441 to 445 # 5, 8, 16, 17, 22, 24
*For # 16 and 17 you need to find the slope of the line before trying to create an equation. What is the rate at which each dog is growing?

8.3: Number of Solutions for a System

When we solve a linear system of two equations in two variables, there are three possibilities.

Intersecting lines   **Parallel lines**   **Coincident lines** live together



Ex. 1: Complete the following table based on the pictures above:

	Slopes (same or different)	Y-intercepts (same or dif)	Number of Solutions
Intersecting lines	different	same or different	1
Parallel lines	same	diff.	0
Coincident lines	same	Same	∞

- We can use this table to help determine how many solutions a system will have!

Ex. 2: Predict the number of solutions for each system of linear equations. Explain your reasoning.

a) ① $y = 2x - 3$
② $y = \frac{1}{2}x + 3$

diff. slope
→ intersecting
$\boxed{\text{1 Solution}}$

b) ① $4x + 10y = 30$
② $2x + 5y = 35$

① $10y = -4x + 30$
$y = -\frac{2}{5}x + 3$
② $5y = -2x + 35$
$y = -\frac{2}{5}x + 7$
→ same slope
→ parallel
$\boxed{\text{No Solution}}$

c) ① $10x - 6y = -12$
② $21y = 42 + 35x$

① $-6y = -10x - 12$
$y = \frac{5}{3}x + 2$
② $y = \frac{5}{3}x + 2$
→ same slope/y-int
→ coincident
∞ Solutions

Ex. 3: Consider the equation $3x - 2y = 3$. Write a second equation to form a linear system with:

$-2y = -3x + 3$

$y = 3/2x - 3/2$

$m = 3/2$

$b = -3/2$

a) only one solution

→ Intersecting

eg. $y = x + 5$

b) no solution

→ Parallel

eg. $y = \frac{3}{2}x + 5$

c) infinitely many solutions

→ Coincident

eg. $y = \frac{3}{2}x - \frac{3}{2}$

Problem Solving- Systems of Equations

Ex. 4: For the school play, one adult ticket cost $5.00 and one student ticket cost $3.00. Twice as many student tickets as adult tickets were sold. The total receipts were $1650. How many of each kind of ticket were sold?

let a=adult
S=student

2×a	s
2× 10 =	20
2× 20 =	40
2× 40 =	80

→ $2a = S$

($) $5a + 3S = 1650$

Isolate a or S

$3S = -5a + 1650$

$S = -5/3a + 550$

→ Graphing Calc.

a s
(150, 300)

There were 150 adults and 300 students.

Ex. 5: A preschool playground has both bicycles and tricycles. There are a total of 30 seats and 70 wheels. How many bicycles are there? How many tricycles are there?

let B = bicycles
T = tricycles

(seats): $B + T = 30$ → $B = 30 - T$

(wheels): $2B + 3T = 70$

→ Isolate B or T

$2B = -3T + 70$

$B = -\frac{3}{2}T + 35$

→ Graphing Calc.

T B
(10, 20)

There are 10 Tricycles and 20 Bicycles.

Homework: ***453 – 458 # 1 – 4, 6, 8, 9, 10a, 13***

p.7

9.1: Solving Systems by Substitution

- A linear system can also be solved by solving one equation for one variable then substituting the result into the other equation.

Ex 1: Solve by substitution

$3x - 4y = -5$ ①
$y + \cancel{2x} = 4$ ②
$\cancel{-2x}$ $-2x$

1. Isolate y in ②

$y = -2x + 4$

2. Substitute into ①

$3x - 4y = -5$

$3x - 4(-2x + 4) = -5$

3. $3x + 8x - 16 = -5$

$11x - \cancel{16} = -5$

$+\cancel{16}$ $+16$

$\frac{\cancel{11}x}{\cancel{11}} = \frac{11}{11}$

$x = 1$

4. $y = -2x + 4$

$y = -2(1) + 4$

$y = 2$

Solution: (1, 2)

STEPS

1. Isolate one variable in one equation. (Find y= or x=)
2. Substitute equation from step 1 into the other equation.
3. Solve for the variable. (there should only be one variable at this point)
4. Once one variable is found, substitute in to either equation to find the second variable.

Ex 2: Solve by substitution

$4x + y = -5$ ①
$2x + 3y = 5$ ②

1. Isolate y in ①

$\cancel{4x} + y = -5$
$\cancel{-4x}$ $-4x$

$y = -4x - 5$

2. Substitute into ②

$2x + 3y = 5$

$2x + 3(-4x - 5) = 5$

3. Solve!

$2x - 12x - 15 = 5$

$-10x - \cancel{15} = 5$
$+\cancel{15}$ $+15$

$\frac{-\cancel{10}x}{-\cancel{10}} = \frac{20}{-10}$

$x = -2$

4. $y = -4x - 5$

$= -4(-2) - 5$

$= 3$

Solution: (-2, 3)

Homework:* Pg. *pages 474 to 475 # 1, 2, 3, 7, 12, 14

9.2: Solving Systems by Elimination

Solving Systems of Equations by Addition/Subtraction (Elimination method)

1. For each pair, add the equations. What do you notice about the resulting equation?

a) $x + y = 4$
$+\ x - y = 2$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{6}{2}$ $x=3$

b) $3x - y = 2$
$+\ -3x + 2y = 6$

$y=8$

c) $-2x + 5y = 10$
$+\ -x - 5y = -4$

$-3x = 6$

→ 1 variable cancels out!

2. For each pair, subtract the second equation from the first. What do you notice about the resulting equation?

a) $3x + 2y = 8$
$-(x + 2y = 3)$

$2x = 5$

b) $2x - 5y = -3$
$-(2x + y = 1)$

$-6y = -4$

c) $2x + y = 5$
$-(3x + y = 1)$

$-x = 4$

→ 1 variable cancels out!

3. Solve $3x - 5y = -9$ ① $+\ 4x + 5y = 23$ ② **by addition/ subtraction (elimination)**

Find intersection (x,y)

$\frac{\cancel{7}x}{\cancel{7}} = \frac{14}{7}$

$x = 2$

Plug in to either equation

① $3(2) - 5y = -9$
$\cancel{6} - 5y = -9$
$-\cancel{6} \quad\quad -6$
$\frac{-\cancel{5}y}{-\cancel{5}} = \frac{-15}{-5}$
$y = 3$

$\boxed{\text{Solution: } (2,3)}$

Not all systems can be solved by adding the two equations. We often must multiply one, or both, of the equations by a constant before one variable can be eliminated. Multiplying by a constant does NOT change the solution.

4. For each pair, by what constant (= number) would you have to multiply the first equation so that the **coefficients of x** are the same in both equations? Perform the multiplication, than subtract the second equation from the resulting equation.

a) $(x + 3y = -7) \times 3$ ①
$3x + 4y = 14$ ②

$$\begin{array}{rl} & 3x + 9y = -21 \quad ① \\ - & (3x + 4y = 14) \quad ② \\ \hline & 5y = -35 \end{array}$$

b) $(2x - y = 5) \times 3$ ①
$8x + 3y = 6$ ②

$$\begin{array}{rl} & 6x - 3y = 15 \quad ① \\ + & 8x + 3y = 6 \quad ② \\ \hline & 14x = 21 \end{array}$$

or =

$$\begin{array}{rl} & 8x - 4y = 20 \\ - & (8x + 3y = 6) \\ \hline & -7y = 14 \end{array}$$

5. Solve ① **3y – 6x = -15** **by elimination**
② $(\mathbf{2y + 3x = 11}) \times 2$

$$\begin{array}{rl} ① & 3y - 6x = -15 \\ + ② & 4y + 6x = 22 \\ \hline & 7y = 7 \end{array}$$

$\frac{\cancel{7}y}{\cancel{7}} = \frac{7}{7}$

$y = 1$

plug into ① or ②

② $2(1) + 3x = 11$

$\cancel{2} + 3x = 11$
$-\cancel{2} \quad\quad -2$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{9}{3}$

$x = 3$

$\boxed{(3, 1)}$

6. Solve $5 \times (\mathbf{2x + 3y = 8})$ **by elimination**
$2 \times (\mathbf{5x - 4y = -6})$

$$\begin{array}{rl} ① & 10x + 15y = 40 \\ ② & -(10x - 8y = -12) \\ \hline & 23y = 52 \end{array}$$

$\frac{\cancel{23}y}{\cancel{23}} = \frac{52}{23}$

$y = \frac{52}{23}$

plug in ① or ②

① $2x + \frac{3}{1}\left(\frac{52}{23}\right) = 8$

$\left(2x + \frac{156}{23} = 8\right) \times 23$

$46x + \cancel{156} = 184$
$-\cancel{156} \quad -156$

$\frac{\cancel{46}x}{\cancel{46}} = \frac{28}{46}$

$x = \frac{28}{46} = \frac{14}{23}$

$\boxed{\left(\frac{14}{23}, \frac{52}{23}\right)}$

Homework: ***pages 488 to 491 # 1, 2, 4, 5a, 9, 10, 11, 15a***

9.3: Comparing all 3 methods

Method	Advantages	Disadvantages
Graphically By Hand Graphing Calculator	-may not be allowed calc. -shows solution clearly if a whole number -shows a picture -shows exact decimal -does some of the work -Can check!	-not always accurate -larger numbers hard to graph -slow (lots of work) -must rearrange (y=)
Algebraic Substitution Elimination (addition/subtraction)	-Can check! -can be faster -can check for mistakes -always work! -simple steps -accurate!	-easy to make mistakes -can be slower -sometimes rearranging needed

Complete pages 492 to 493 # 1 to 3 in the space below.

Homework: ***pages 498 to 501 # 1, 2, 3, 6, 7, 8***