Notes

November-17-15
11:29 AM

8.1: Solving Systems of Equations Graphically

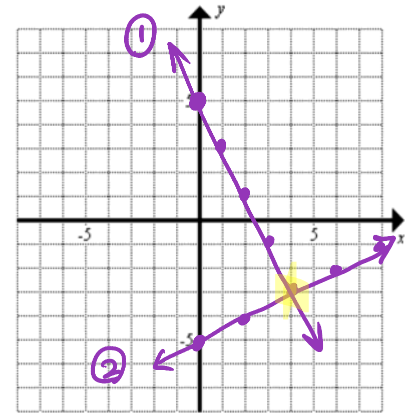
Recall from Math 10 that a **system of equations** consists of 2 or more equations considered together simultaneously. Any solution must satisfy each equation of the system.

- Graphically, the **intersection** corresponds to the (x,y) coordinates that satisfy each equation.

Ex. 1: Solve by graphing by hand:

$$\begin{cases} 2x + y = 5 & ① \\ x - 2y = 10 & ② \end{cases}$$

$y = mx + b$



① $2x + y = 5$

$y = \frac{-2}{1}x + 5$

② $x - 2y = 10$

$\frac{-2y}{-2} = \frac{-x+10}{-2}$

$y = \frac{1}{2}x - 5$

Linear-Linear System

$\boxed{\text{Solution: } (4,-3)}$

Verify your solutions by plugging into the original equations:

① $2(4) + (-3) = 5$
$8 - 3 = 5$ ✓

② $4 - 2(-3) = 10$
$4 + 6 = 10$ ✓

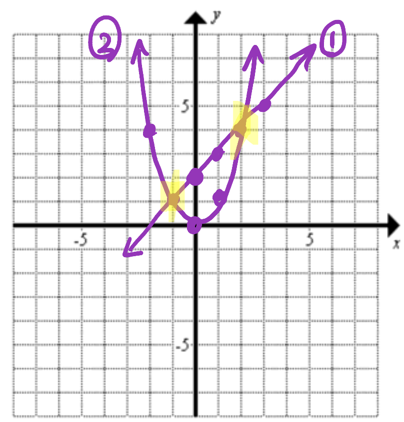
The above is an example of a system of **linear-linear equations**.
We will also be considering **linear-quadratic** systems and **quadratic-quadratic** systems.

Ex. 2: Solve by graphing by hand:

$$\begin{cases} y = x + 2 & ① \\ y = x^2 & ② \end{cases}$$

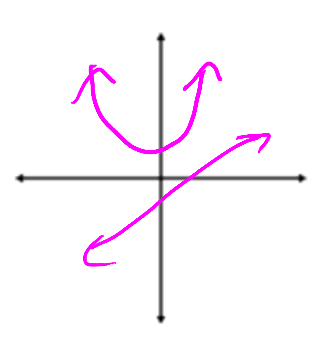
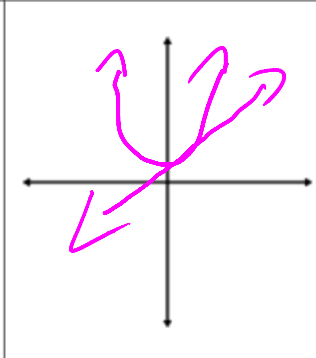
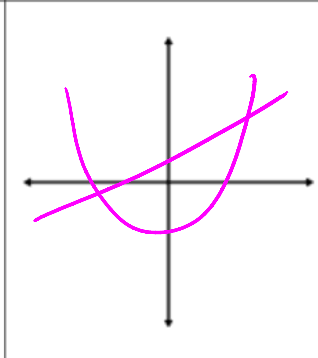
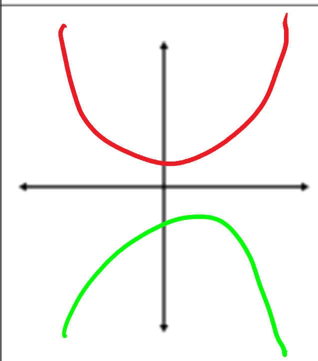
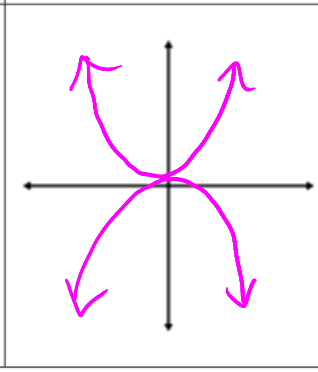
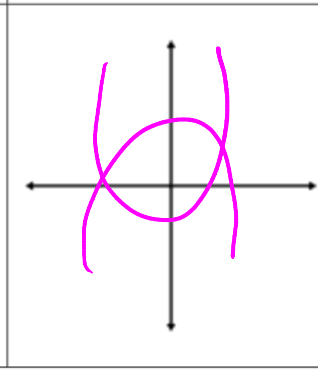
Linear-Quadratic

vertex: (0,0)
stretch = 1



$\boxed{\text{Solutions: } (-1,1) \text{ and } (2,4)}$

Ex. 3: Complete the following table. Draw a possible diagram for the number of solution(s).

	NO SOLUTION	1 SOLUTION	2 SOLUTIONS
Linear-Quadratic System			
Quadratic-Quadratic System			

Ex. 4: Solve the following system using technology. Verify your solution(s).

$$\begin{cases} 2x^2 - 16x - y = -35 & ① \\ 2x^2 - 8x - y = -11 & ② \end{cases}$$

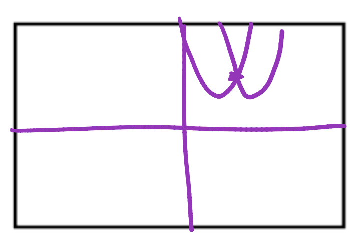
Quadratic-Quadratic

① $2x^2 - 16x - y = -35$

$2x^2 - 16x + 35 = y$

② $2x^2 - 8x - y = -11$

$2x^2 - 8x + 11 = y$



x [-10, 10] y [-10, 10]

$Y_1 = 2x^2 - 16x + 35$

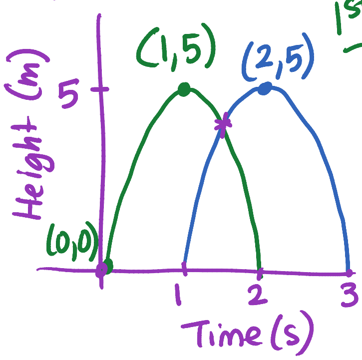
$Y_2 = 2x^2 - 8x + 11$

$\boxed{\text{Solution: } (3, 5)}$

Ex. 5: In a Cirque du Soleil stunt, performers are launched toward each other from two slightly offset seesaws. The first performer is launched, and 1 s later the second performer is launched in the opposite direction. They both perform a flip and give each other a high five in the air. Each performer is in the air for 2 s. The height above the seesaw versus time for each performer during the stunt is approximated by a parabola as shown on pg. 432 in the textbook.

(a) Determine the system of equations that models the performers' height during the stunt.
(b) Solve the system graphically using technology.
(c) What is the meaning of your solution?

p.432



a) $y=a(x-p)^2+q$

1st performer:

$y=a(x-1)^2+5$
$0=a(0-1)^2+5$
$0=a+5$
$-5=a$

$\boxed{y=-5(x-1)^2+5}$

2nd performer:
→same stretch

$\boxed{y=-5(x-2)^2+5}$

system!

b) →use Graphing Calc.
(1.5, 3.75)

c) After 1.5 s, each 3.75 m high.

8.2: Solving Systems of Equations Algebraically (day 1)

Recall from Math 10 that there are 2 methods to solve a system of equations algebraically:

1. SUBSTITION

   Isolate one variable in one equation.
   Substitute this into the other equation and solve.
   Plug your solution into the first equation and solve for the remaining variable.

2. ELIMINATION (Addition/Subtraction)

   Multiply/Divide the equations by a number to match one of the variable coefficients.
   Add/Subtract the equations together to eliminate that variable and solve.
   Plug your solution into the first equation and solve for the remaining variable.

* In both cases we must **CHECK** our solution.
In this course we will use the same methods to solve **linear-quadratic** and **quadratic-quadratic** systems.

Ex.1: Solve the following system of equations using both methods:

$$\begin{cases} x+2y=-3 & ① \\ 2x+3y=-4 & ② \end{cases}$$

Substitution

① $x = -3-2y$

Substitute into ②

② $2x+3y=-4$

$2(-3-2y)+3y=-4$

$-6-4y+3y=-4$

$-6-y=-4$

$-2=y$

$x=-3-2y$

$=-3-2(-2)$

$=1$

$(1,-2)$

Elimination

$(x+2y=-3)\times 2$ ①

$2x+3y=-4$ ②

$-(2x+4y=-6)$

$-y=2$

$y=-2$

① $x+2(-2)=-3$

$x-4=-3$

$x=1$

$(1,-2)$

Ex.2: Solve the following system of equations using both methods:

Linear-Quadratic

$$\begin{cases} 5x-y=10 & ① \\ x^2+x-2y=0 & ② \end{cases}$$

Substitution

① $5x-10=y$

② $x^2+x-2(5x-10)=0$

$x^2+x-10x+20=0$

$x^2-9x+20=0$

Quad. Eq.!

$(x-4)(x-5)=0$

$x=4,5$

$x_1=4$

$y_1=5(4)-10$

$=10$

$x_2=5$

$y_2=5(5)-10$

$=15$

$\boxed{(4,10) \text{ and } (5,15)}$

Elimination

$(5x-y=10)\times 2$ ①

$x^2+x-2y=0$ ②

$-(10x-2y=20)$

$x^2-9x=-20$

$x^2-9x+20=0$

$x=4,5$

⋮

YOUR TURN

Solve the following system of equations using both methods:

$$\begin{cases} 3x + y = -9 & ① \\ 4x^2 - x + y = -9 & ② \end{cases}$$

① $y = -3x - 9$

② $4x^2 - x + (-3x - 9) = -9$

$4x^2 - 4x = 0$ ← Factor! GCF $= 4x$

$4x(x-1) = 0$

$x_1 = 0$ $\quad$ $x_2 = 1$

$y_1 = -3(0) - 9$ $\quad$ $y_2 = -3(1) - 9$

$= -9$ $\quad$ $= -12$

$\boxed{(0,-9) \text{ and } (1,-12)}$

$$\begin{array}{r} - \quad 3x + y = -9 \\ (4x^2 - x + y = -9) \\ \hline -4x^2 + 4x = 0 \\ -4x(x-1) = 0 \\ \vdots \end{array}$$

Ex. 3: Solve the following system using either method. State your chosen method.

Quad.-Quad. $\quad$ Elimination

$$\begin{array}{r} \begin{cases} 3x^2 - x - y + \cancel{2} = \cancel{0}\ 2 & ① \\ -(6x^2 + 4x - y = 4) & ② \end{cases} \\ \hline -3x^2 - 5x = -2 \end{array}$$

$0 = 3x^2 + 5x - 2$ $\quad$ (-6)

$0 = \underbrace{3x^2 + 6x}\, - 1x - 2$

$0 = 3x(x+2) - 1(x+2)$

$0 = (x+2)(3x-1)$

$x = -2, \frac{1}{3}$

$x_1 = -2$

② $6(-2)^2 + 4(-2) - y = 4$

$24 - 8 - 4 = y$

$12 = y_2$

$(-2, -12)$

$x_2 = \frac{1}{3}$

② $6\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right) - y = 4$

$\frac{2}{3} + \frac{4}{3} - 4 = y$

$2 - 4 = y$

$-2 = y$

$\left(\frac{1}{3}, -2\right)$

8.2: Solving Systems of Equations Algebraically (day 2)

Today we will be extending the same techniques from last day to solve more complex word problems.

Ex.1: Determine two integers such that the sum (+) of the smaller number and twice the larger number is 46. Also, when the square of the smaller number is decreased (−) by three times the larger, the result is 93.

let: x = smaller #, y = larger #

a) Write a system of equations for this problem.
b) Solve the system algebraically. ← sub. or elim.

a) $(x + 2y = 46) \times 3$
$(x^2 - 3y = 93) \times 2$

b) $3x + 6y = 138$
$+ (2x^2 - 6y = 186)$
$2x^2 + 3x = 324$

$2x^2 + 3x - 324 = 0$

$x = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(-324)}}{2(2)}$

$= \dfrac{-3 \pm 51}{4}$

$= 12, \cancel{-13.5}$

Not Integer!

① $12 + 2y = 46$
$2y = 34$
$y = 17$

$\boxed{\text{2 Integers are 12 and 17}}$

Ex.2: A Canadian cargo plane drops a crate of emergency supplies to aid-workers on the ground. The crate drops freely at first before a parachute opens to bring the crate gently to the ground. The crate's height, h, in meters, above the ground t seconds after leaving the aircraft is given by the following two equations.

$h = -4.9t^2 + 700$ represents the height of the crate during the free fall
$h = -5t + 650$ represents the height of the crate with the parachute open

a) How long after the crate leaves the aircraft does the parachute open? (Nearest hundredth)

Intersection:
(3.75, 631.27)
After 3.75 s

b) What height above the ground is the crate when the parachute opens? (Nearest tenth)

631.27 m

c) Verify your solution.
① $631.27 = -4.9(3.75)^2 + 700$ ✓
② $631.27 = -5(3.75) + 650$ ✓