Notes

September-29-15
8:07 AM

3.1: Investigating quadratic functions in vertex form (Day 1)

Objectives:

- Define a quadratic function.
- Determine the effects of "p" and "q" on the graph of $y = a(x-p)^2 + q$

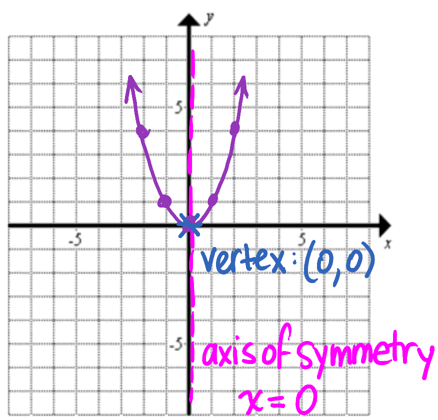
A **quadratic function** is a function of degree 2. (recall: **degree** = highest exponent value).

for example: $y = x^2$, $f(x) = -\frac{1}{2}x^2 + 5x - 3$, $g(x) = (x-5)(x+4)$

In general, a quadratic function is of the form: $\boxed{y = ax^2 + bx + c}$ where $a \neq 0$

'Basic Quadratic Function'

Ex. 1: Graph the function $y = x^2$ using a table of values.



x	y
−2	4
−1	1
0	0
1	1
2	4

What is the **domain** of this graph? all x values

x can be all real numbers.

$x \in \mathbb{R}$

What is the **range** of this graph? all y values

y can be greater than or equal 0

$y \geq 0$

Definitions:

- **Parabola**: the *shape* of the graph of a quadratic equation
- **Vertex**: the highest (maximum) or lowest (minimum) point of the parabola
- **Axis of symmetry**: the equation of the vertical line that passes through the vertex (note: vertical line equations are always of the form x = p)

*** Always graph at least 5 points clearly when graphing a parabola ***

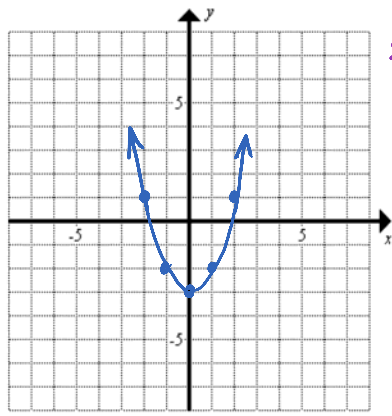
Graphing: $y = x^2 + q$:

The "q" value is the *vertical shift* of the parabola $y = x^2$
To graph, simply shift the graph of $y = x^2$ up or down by the value of "q"

Ex. 2: Graph $y = x^2 - 3$

1) Graph 'Basic' Parabola

2) Shift Down 3



State the:

a) vertex: $(0,-3)$

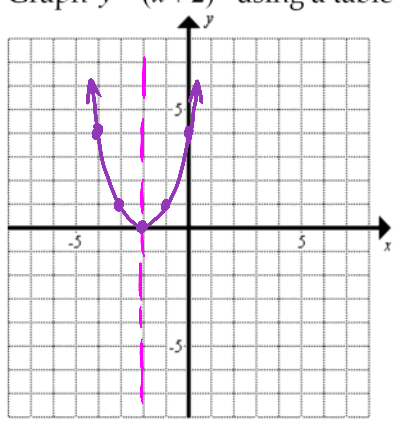
b) equation of the axis of symmetry: $x=0$

c) domain: $x \in \mathbb{R}$

d) range: $y \geq -3$

Graphing $y = (x-p)^2$:

Ex. 3: Graph $y = (x+2)^2$ using a table of values.



x	y
-4	4
-3	1
-2	0
-1	1
0	4

State the:

a) vertex: $(-2,0)$

b) equation of the axis of symmetry: $x=-2$

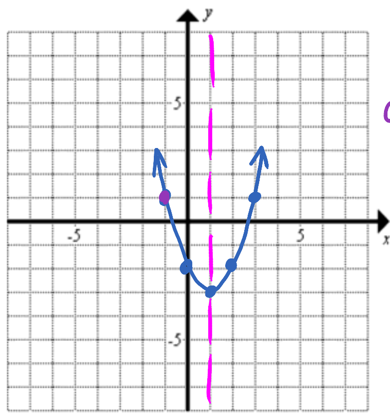
c) domain: $x \in \mathbb{R}$

d) range: $y \geq 0$

The "p" value is the *horizontal shift* of the parabola $y = x^2$
To graph, simply shift the graph of $y = x^2$ left or right by the **opposite** value of p.

$p=1$ $q=-3$

Ex. 4: Graph $y=(x-1)^2-3$ by comparing to the graph of $y=x^2$



$p=1$
→shift right 1
$q=-3$
→shift down 3

State the:

a) vertex: $(1,-3)$

b) equation of the axis of symmetry: $x=1$

c) domain: $x \in \mathbb{R}$

d) range: $y \geq -3$

left/right opposite

$y=(x-p)^2+q$ ← up/down

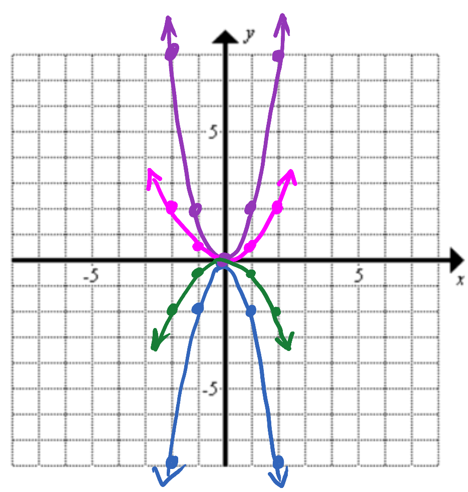
3.1: Investigating quadratic functions in vertex form (Day 2)

Objectives:

- Determine the effects of "a" on the graph of $y=a(x-p)^2+q$
- Graph any quadratic function in vertex form $y=a(x-p)^2+q$

Ex. 1: Graph the following on the same grid using a table of values.

x	$y=2x^2$	$y=\frac{1}{2}x^2$	$y=-2x^2$	$y=-\frac{1}{2}x^2$
-2	8	2	-8	-2
-1	2	0.5	-2	-0.5
0	0	0	0	0
1	2	0.5	-2	-0.5
2	8	2	-8	-2



- What effect does the value of "a" have on the graph?
  - open up or down
  - wider or thinner

If a > 0 (+): the parabola opens __up__ → ◡

If a < 0 (−): the parabola opens __down__ → ◠

The value of a is the "stretch" of the parabola.

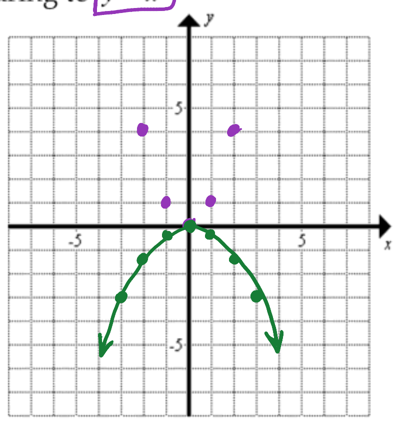
We graph $y = ax^2$ by multiplying the values of $y = x^2$ ("Basic") by "a"

$y = x^2$		$y = ax^2$	
x	y	x	y
0	0	0	0
1	1	1	a
2	4	2	4a
3	9	3	9a

×a

For our "new" parabol we graph starting fron the vertex and shifting over (x) and up/down the original y times "a'

Ex. 2: Graph $y = -\frac{1}{3}x^2$ by comparing to $\boxed{y = x^2}$



$y = -\frac{1}{3}x^2$

p=0 q=0

→ no shift: vertex (0,0)

$a = -\frac{1}{3}$ → open down

	$\times -\frac{1}{3}$
1	$1 \times -1/3 = -1/3$
2	$4 \times -1/3 = -4/3$
3	$9 \times -1/3 = -9/3 = -3$

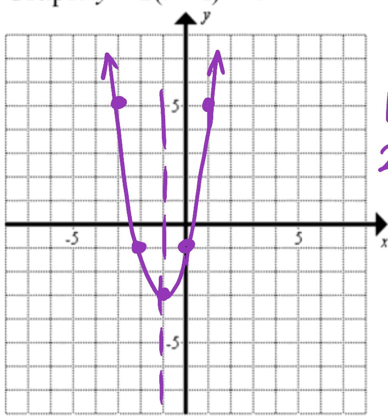
In general for any parabola in vertex form $y = a(x-p)^2 + q$

- vertex: (p, q) * note sign change on "p"
- equation of the axis of symmetry: x = p

To graph: $y = a(x-p)^2 + q$

1. Graph vertex (p, q)
2. Graph 2 points on either side of the vertex using the stretch value of "a" compared to $y = x^2$. *Make table.

Ex. 3: Graph $y = 2(x+1)^2 - 3$



$a=2$
$p=-1$
$q=-3$

1. vertex: $(-1,-3)$

2.

	×a
1	1×2 = 2
2	4×2 = 8
3	9×2 = 18

*Move from vertex

State the:

a) vertex: $(-1,-3)$

b) equation of the axis of symmetr $x=-1$

c) domain: $x \in \mathbb{R}$

d) range: $y \geq -3$

Ex. 4: Determine the quadratic function written in the ~~standard~~ vertex form $y = a(x-p)^2 + q$ with the given information:

a) vertex (3, – 5) , and $a = 2$

p q

$y = 2(x-3)^2 - 5$

b) congruent to $y = 0.4x^2$, opens down, and vertex (3, -3)

same shape

p q

$a = -0.4$

$y = -0.4(x-3)^2 - 3$

c) vertex (0, 3), passing through (3, 4)

p q

$y = a(x-0)^2 + 3$
$y = a(x)^2 + 3$
→plug in (3,4), solve for a.

$4 = a(3)^2 + 3$
$4 = 9a + 3$
$1 = 9a$
$a = \frac{1}{9}$

$\boxed{y = \frac{1}{9}(x)^2 + 3}$

d) vertex (4, –1), and y – intercept 3 → (0,3)

p q

$y = a(x-4)^2 - 1$
→plug in (0,3), solve for a
$3 = a(0-4)^2 - 1$
$3 = 16a - 1$

$4 = 16a$
$a = \frac{4}{16} = \frac{1}{4}$

$\boxed{y = \frac{1}{4}(x-4)^2 - 1}$

Ex.5: Write the equation from Example 3, part a, in general form ($y = ax^2 + bx + c$)

$y = 2(x+1)^2 - 3$
$= 2(x+1)(x+1) - 3$
$= 2(x^2 + x + x + 1) - 3$
$= 2(x^2 + 2x + 1) - 3$
$= 2x^2 + 4x + 2 - 3$

$\boxed{y = 2x^2 + 4x - 1}$

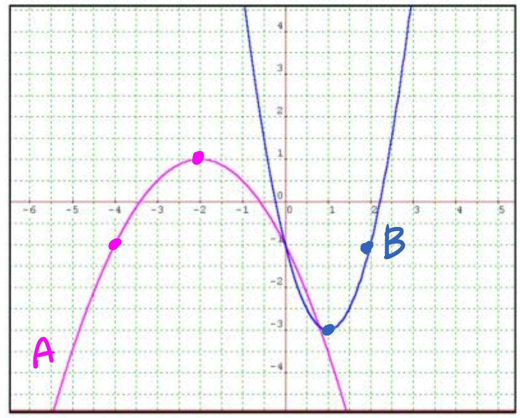
3.2: Investigating quadratic functions in general form

$y = ax^2 + bx + c$

Objectives:

- Determine the vertex, axis of symmetry, domain and range for quadratic functions in general form.
- Use a graphing calculator to graph quadratic functions.
- Analyze word problems involving quadratic functions.

Ex. 1: What are the equations of the functions shown?



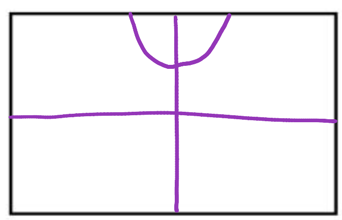
→ extra practice for the quiz!

A) $y = a(x+2)^2 + 1$
$-1 = a(-4+2)^2 + 1$
$-2 = 4a$
$a = -\frac{1}{2}$
$\boxed{y = -\frac{1}{2}(x+2)^2 + 1}$

B) $y = a(x-1)^2 - 3$
$-1 = a(2-1)^2 - 3$
$2 = a$
$\boxed{y = 2(x-1)^2 - 3}$

Part 1. Graph each of the following functions using a graphing calculator. For each function, determine the coordinates of the vertex and any intercepts. Round to the nearest tenth, where necessary for 1 to 4 and to the nearest hundredth for 5.

1. $y = x^2 + 5$
   vertex: $(0,5)$ ← 2nd TRACE 3: minimum
   the y-intercept: $(0,5)$ 1: value → $x=0$
   the x-intercepts: none

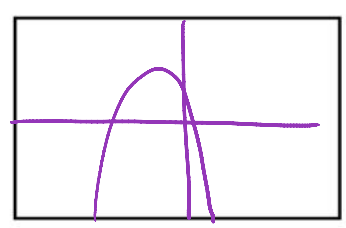


$Y_1 = x^2 + 5$

window settings → x [-10, 10] y [-10, 10]

screen display

2. $y = -x^2 - 2x + 5$
   vertex: $(-1,6)$
   the y-intercept: $(0,5)$ or $= 5$
   the x-intercepts: $(-3.45, 0) = -3.45$
   ["zero"] $(1.45, 0)$ 1.45



$Y_1 = -x^2 - 2x + 5$

x [-10, 10] y [-10, 10]

6

3. $y + 7x = -0.5x^2 - 3$

   vertex: $(-7, 21.5)$

   the y-intercept: $(0,-3)$ or $= -3$

   the x-intercepts: $(-13.56, 0) = -13.56$

   $(-0.44, 0) = -0.44$

$Y_1 = -0.5x^2 - 7x - 3$

x [-50, 50] y [-50, 50]

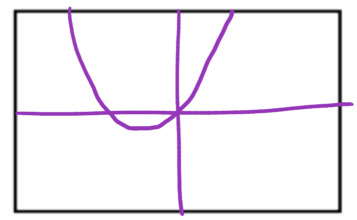
4. $y = 0.1x(5 + x)$

   vertex: $(-2.5, -0.625)$

   the y-intercept: $(0,0)$ or $= 0$

   the x-intercepts: $(-5, 0)$ or $= -5$

   $(0,0)$ or $= 0$



$Y_1 = 0.1x(5 + x)$

x [-20, 20] y [-20, 20]

Applications of Quadratic Functions

Part 2. Problem Solving. Solve using a graphing calculator. Round answers to the nearest hundredth.

1. A rock is thrown off a cliff. The height, in metres, with respect to time, in seconds, is defined by the quadratic function $h(t) = -4.9t^2 + 90t + 50$.

   a) What is the maximum height? 463.27 m

   ↳ vertex! $(9.18, 463.27)$

   b) When does it reach this height? 9.18 s

   c) How long does it take to reach the ground? $(18.91, 0)$ 18.91 s

   x-int = 'zero'

   d) How high is the cliff? 50m

   y-int → value $x=0$ $(0, 50)$

   e) What is the domain of this function? $0 \le t \le 18.91$

2. Two numbers have a difference of 10. Their product is a minimum. Determine the two numbers.

(−) →vertex!

let x, y = 2 numbers

$x - y = 10$

$\Rightarrow x = y + 10$

Product: $P = xy$

$P = (y+10)y$

$P = y^2 + 10y$

→vertex: $(-5, -25)$

y P

If $y = -5$

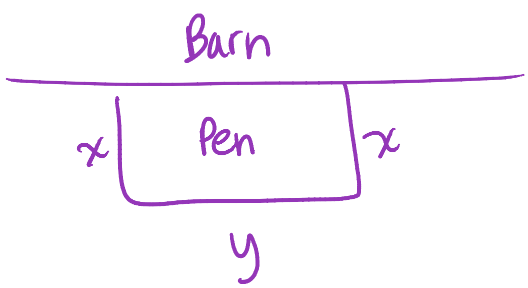
$x = y + 10$

$= -5 + 10$

$= 5$

$\boxed{\text{The 2 numbers are } -5 \text{ and } 5}$

3. A rectangular pen is to be built along the side of a barn to house chickens.



Fence: $2x + y = 60$

$\rightarrow y = 60 - 2x$

Area: $A = xy$

$A = x(60 - 2x)$

$A = 60x - 2x^2$

→vertex: $(15, 450)$

x A

→vertex!

a) Find the maximum area that can be enclosed with 60 m of fencing if the barn is one side of the enclosure.

$\boxed{\text{Area} = 450\,m^2}$

b) What are the dimensions that gives the maximum area?

$x = 15\,m$

$y = 60 - 2x$

$= 60 - 2(15)$

$= 30\,m$

Dimensions are 15 m by 30 m

3.3: Completing the Square

Objectives:

- Converting quadratic functions from standard to vertex form.
- Writing quadratic functions to model situations.

Given a quadratic function in standard form $y = ax^2 + bx + c$ we don't know how to graph by hand.

"square"

- We can RE-WRITE the standard form into vertex form $y = a(x-p)^2 + q$ by the method of **completing the square.**

Ex. 1: Rewrite $y = x^2 + 6x + 8$ in vertex form.

$$\left[\tfrac{1}{2}(6)\right]^2 = 9$$

$$y = (x^2 + 6x) + 8$$
$$= (x^2 + 6x + 9 - 9) + 8$$
$$= \underbrace{(x^2 + 6x + 9)}_{\text{factor!}} - 9 + 8$$
$$= (x+3)^2 - 1$$

→ vertex: $(-3, -1)$

- Why do you think this is called **completing the square**? Where is the **square**?

Ex. 2: Rewrite $y = 3x^2 - 12x + 11$ in vertex form.

$$y = a(x-p)^2 + q$$

$$\left[\tfrac{1}{2}(-4)\right]^2$$

$$= 3(x^2 - 4x) + 11$$
$$= 3(x^2 - 4x + 4 - 4) + 11$$
$$= 3\underbrace{(x^2 - 4x + 4)}_{\text{factor!}} - 12 + 11$$
$$= 3(x-2)^2 - 1$$

→ vertex: $(2, -1)$

<u>To complete the square:</u>

1. Group the x terms - factor out 'a' from the two terms with x if necessary.
2. Add and subtract $(\frac{1}{2}b)^2$ inside of the brackets from part 1.
3. Move the subtracted value outside of the brackets by multiplying by 'a'.
4. Factor x terms inside of brackets and simplify constant terms outside of the brackets.

Ex. 3: Analyze the function $y=-2x^2+3x-2$ by re-writing in vertex form. State the vertex, axis of symmetry, maximum or minimum and domain/range.

$$y = \frac{-2x^2}{-2} + \frac{3x}{-2} - 2$$

$$= -2\left(x^2 - \frac{3}{2}x\right) - 2$$

$$\left[\frac{1}{2}\left(-\frac{3}{2}\right)\right]^2 = \left(-\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$= -2\left(x^2 - \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}\right) - 2$$

$$-\frac{9}{16}(-2) = \frac{18}{16}$$

$$= -2\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) + \frac{9}{8} - \frac{16}{8}$$

half

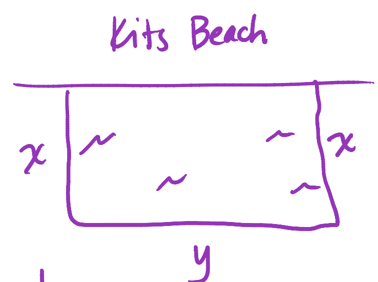
$$\boxed{= -2\left(x - \tfrac{3}{4}\right)^2 - \tfrac{7}{8}}$$

vertex: $(3/4, -7/8)$ domain: $x \in \mathbb{R}$

axis: $x = 3/4$ range: $y \le -7/8$

Max/min: $= -7/8$

Ex. 4: We will be roping off a swimming area down at Kits Beach for the day. If we have only 100 m of rope and want a **maximized** swimming <u>area</u>, what should the dimensions of the area be?



Rope: $2x + y = 100$

$y = 100 - 2x$

Area: $A = xy$

$= x(100 - 2x)$

$= 100x - 2x^2$ → vertex? complete the square!

$= -2x^2 + 100x$

$= -2(x^2 - 50x)$ $\left(\frac{1}{2}(-50)\right)^2$

$= -2(x^2 - 50x + 625 - 625)$

$= -2(x^2 - 50x + 625) + 1250$

$A = -2(x - 25)^2 + 1250$

vertex: $(25, 1250)$

x ↗ ↖ A

$\boxed{\text{Dimensions: 25m by 50m}}$

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