# Notes

October-22-15
9:50 AM

# 4.1: Graphical Solutions of Quadratic Equations

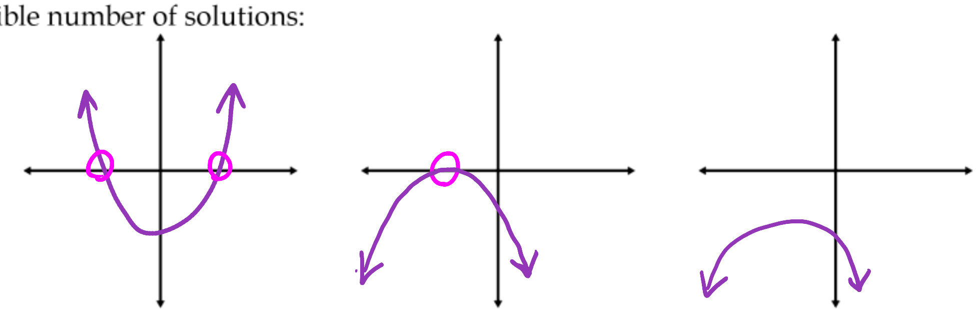
**Objectives:**

- Determine the **roots** and **zeros** of a quadratic equation by graphing.

equation: can solve

"Solutions" of any quadratic equation $ax^2+bx+c=0$ are called **zeros** or **roots**. Graphically, they correspond to the x-intercepts (where y = 0).

Possible number of solutions:



2 solutions     1 solution     No Solution

There are 2 methods to solving quadratic equations graphically:

METHOD #1:

- We must first move everything over to one side of the equation so it equals 0.
- We will then find the intersection of the graph with the line y = 0. (This is called finding the roots or zeros!)

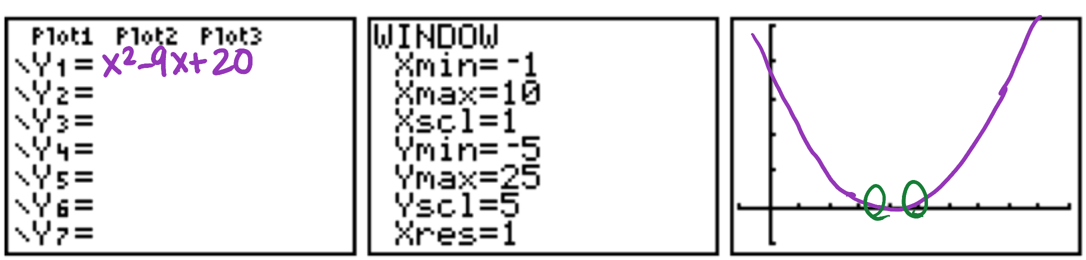
Ex. 1: Solve $x^2-9x+25=5$ by graphing.

$-5 \quad -5$

Move everything over to one side. What equation do you get? $x^2-9x+20=0$ Finding "zeros"

Now, enter that equation into the Y= screen and GRAPH (use the WINDOW settings below)

*Fill in the screens below:*




Set the WINDOW to these settings and GRAPH!

Now, use CALC ($2^{nd}$ TRACE) to find the *zeros*: ( 4 , 0 ) and ( 5 , 0 )

Solutions are: $x = 4, 5$

$x^2 - 9x + 20 = 0$

Check!

$(4)^2 - 9(4) + 20 = 0$ $(5)^2 - 9(5) + 20 = 0$

$16 - 36 + 20 = 0$ ✓ $25 - 45 + 20 = 0$ ✓

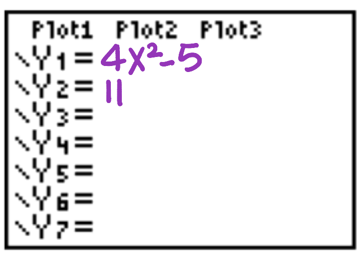
METHOD #2:

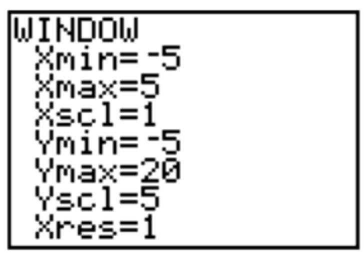
- Graphing each side of equation and finding the intersection (like a system of equations!)
- Enter the left side of the equation into $Y_1$ and enter the right side of the equation into $Y_2$
- Graph and then use CALC to find the intersection point

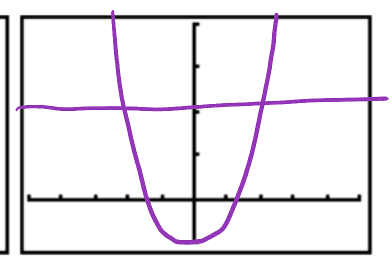
Ex. 3: Solve $\underbrace{4p^2 - 5}_{Y_1} = \underbrace{11}_{Y_2}$ by graphing a related system.

→ 2 equations!

*Fill in the screens below:*



Set the WINDOW to these settings and GRAPH!

Now, use CALC ($2^{nd}$ TRACE) to find the intersection points: ( −2 , 11 ) , ( 2 , 11 )

Solutions: $p = -2, 2$

**Practice Exercises:** Use the methods outlined above to solve the following (try 2 of each!)

1. $x^2 - 10x + 24 = 0$

   $\boxed{x = 4, 6}$

2. $8b^2 = 49 + b^2$

   $\boxed{b = -2.65, 2.65}$

3. $x^2 + 6x - 15 = 4x$

   $\boxed{x = -5, 3}$

4. $3.5x^2 + 16x - 42 = 3$

   $\boxed{x = -6.54, 1.97}$

## 4.2a: Solving Quadratic Equations by Factoring

**Objectives:**

- Factor different types of quadratic expressions
- Solving quadratic equations by factoring

Recall that a quadratic equation is an equation that can be written in the form: $ax^2 + bx + c = 0$
where a, b and c are constants and $a \neq 0$

As with last class, we can "solve" a quadratic equation by setting the equation equal to zero and finding the **roots** or **zeros** of the equation. We can do this by factoring.

- How many possible solutions might we have?

2 or 1 or 0

- How can we check any solution(s)?

Plug in to the x at the beginning.

*Set = 0

Solve the following quadratic equations by factoring. Check your solution(s).

1) $(x - 5)(x + 2) = 0$

$x-5=0$ $\quad$ $x+2=0$

$\boxed{x=5}$ $\quad$ $\boxed{x=-2}$

Check:

$(5-5)(5+2)=0$ ✓

$(-2-5)(-2+2)=0$ ✓

2) $9x^2 = 16$

Diff. of squares!

$9x^2-16=0$

$(3x+4)(3x-4)=0$

$3x+4=0$ $\quad$ $3x-4=0$

$\boxed{x=-4/3}$ $\quad$ $\boxed{x=4/3}$

3) $4y^2 - 8 = 8$

$4y^2-16=0$

$4(y^2-4)=0$

$4(y+2)(y-2)=0$

$\boxed{y=-2,2}$

4) $-4m^2 + 24m = 0$

$-4m(m-6)=0$

$-4m=0$ $\quad$ $m-6=0$

$\boxed{m=0}$ $\quad$ $\boxed{m=6}$

5) $x^2 - 9x + 20 = 0$

$(x-5)(x-4)=0$

$x-5=0$ $\quad$ $x-4=0$

$\boxed{x=5}$ $\quad$ $\boxed{x=4}$

Check:

$(5)^2-9(5)+20=0$ ✓

$(4)^2-9(4)+20=0$ ✓

6) $6x^2 - 17x + 12 = 0$

72 $\quad$ $-9,-8$

"Decomposition"

$6x^2-9x-8x+12=0$

$3x(2x-3)-4(2x-3)=0$

$(2x-3)(3x-4)=0$

$2x-3=0$ $\quad$ $3x-4=0$

$\boxed{x=\frac{3}{2}}$ $\quad$ $\boxed{x=\frac{4}{3}}$

## 4.2b: Solving (harder) Quadratic Equations by Factoring

Warm-Up:

Solve each of the following by factoring:

a) $0 = x^2 + 10x + 21$

$0 = (x+7)(x+3)$

$x+7=0 \quad x+3=0$

$\boxed{x=-7,-3}$

b) $5x^2 - 20 = 0$

$5(x^2-4)=0$

$5(x+2)(x-2)=0$

$\boxed{x=-2,2}$

c) $8x^2 = -5 + 14x$ 40

$8x^2 - 14x + 5 = 0$

$8x^2 - 10x - 4x + 5 = 0$

$2x(4x-5) - 1(4x-5) = 0$

$(4x-5)(2x-1)=0$

$4x-5=0 \quad 2x-1=0$

$\boxed{x=\frac{5}{4}, \frac{1}{2}}$

d) $\frac{1}{4}x^2 + 2x - 5 = 0$

$\frac{1}{4}(x^2+8x-20)=0$

$\frac{1}{4}(x+10)(x-2)=0$

$\boxed{x=-10,2}$

Ex. 1 : Solve by factoring $12(x+2)^2 + 24(x+2) + 9 = 0$. Check your solution(s).

Instinct: Simplify. IGNORE THIS!

let $x+2=z$

$\frac{12z^2}{3} + \frac{24z}{3} + \frac{9}{3} = \frac{0}{3}$ GCF=3

$4z^2 + 8z + 3 = 0$ 12

$4z^2 + 6z + 2z + 3 = 0$

$2z(2z+3) + 1(2z+3) = 0$

$(2z+3)(2z+1) = 0$

$2z+3=0 \quad 2z+1=0$

$z=-\frac{3}{2} \quad z=-\frac{1}{2}$

But! $x+2=z \rightarrow x=z-2$

$x=-\frac{3}{2}-\frac{4}{2} = \boxed{-\frac{7}{2}}$

$x=-\frac{1}{2}-\frac{4}{2} = \boxed{-\frac{5}{2}}$

Your Turn: Solve ~~$12(x+2)^2 + 24(x+2) + 9 = 0$~~

$-2(n+3)^2 + 12(n+3) + 14 = 0$

let $a = n+3$

$\frac{-2a^2}{-2} + \frac{12a}{-2} + \frac{14}{-2} = \frac{0}{-2}$

$a^2 - 6a - 7 = 0$

$(a-7)(a+1) = 0$

$a = 7, -1$

$\rightarrow n+3=a$

$\rightarrow n=a-3$

$n=7-3 \quad n=-1-3$

$\boxed{n=4} \quad \boxed{n=-4}$

Ex. 2: Write a quadratic equation whose roots are $-\frac{1}{2}$ and $\frac{4}{3}$ in standard form $ax^2+bx+c=0$

$2 \times x = -\frac{1}{\cancel{2}} \times \cancel{2}$ $\quad 3 \times x = \frac{4}{\cancel{3}} \times \cancel{3}$

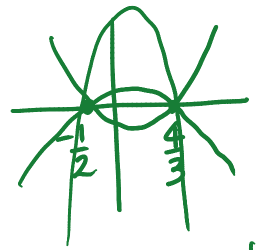
$2x=-1 \quad 3x=4$

$2x+1=0 \quad 3x-4=0$

$(2x+1)(3x-4)=0$

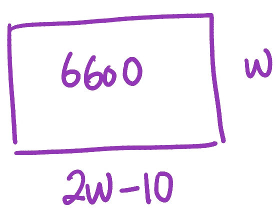
$6x^2-8x+3x-4=0$

$\boxed{6x^2-5x-4=0}$




any stretch is ok!

Ex. 3: The length of an outdoor lacrosse field is 10 m less than twice the width. The area of the field is 6600 $m^2$. Determine the dimensions of the field.




Area = w l

$6600 = w(2w-10)$

$6600 = 2w^2-10w$

$\frac{0}{2} = \frac{2w^2}{2} - \frac{10w}{2} - \frac{6600}{2}$

$0 = w^2-5w-3300$

$0 = (w-60)(w+55)$

$w = 60, \cancel{-55}$

not possible! (width ≮ 0)

Dimensions:
60 m by 110 m

## 4.3: Solving quadratic equations by completing the square

or possible!

Sometimes factoring quadratic equations is not practical. We can use the method of **completing the square** from Chapter 3 to help us find the zeros and roots.

Ex. 1: Solve and check your solutions(s).

Form: $(x-p)^2+q=0$

$(x-p)^2=-q$

$(x-1)^2-49=0$

$\sqrt{(x-1)^{\cancel{2}}} = \pm\sqrt{49}$

$x-1=\pm 7$

$x = \pm 7+1 \rightarrow x_1 = +7+1 = 8$

$x_2 = -7+1 = -6$

Check!

$(8-1)^2-49=0$ ✓

$(-6-1)^2-49=0$ ✓

5

Ex. 2: Solve $x^2 - 21 = -\cancel{10x}$ by completing the square. Express your answers to the nearest tenth.

$\left[\frac{1}{2}(10)\right]^2$

$+10x \quad +\cancel{10x}$

$(x^2 + 10x) - 21 = 0$ ← Does not factor!

$(x^2 + 10x + 25 - 25) - 21 = 0$

$(x^2 + 10x + 25) - 25 - 21 = 0$

$(x+5)^2 - 46 = 0$

$\sqrt{(x+5)^2} = \pm\sqrt{46}$

$x + 5 = \pm\sqrt{46}$

$\boxed{x = \pm\sqrt{46} - 5}$ ← exact value

$x_1 = \sqrt{46} - 5 = \boxed{1.8}$

$x_2 = -\sqrt{46} - 5 = \boxed{-11.8}$

Solving quadratic equations by completing the square:

1. Set the equation equal to zero and complete the square (follow the steps in section 3.3).
2. Isolate the squared term.
3. Take the positive and negative square root. $\pm$
4. Solve the 2 corresponding equations for x.

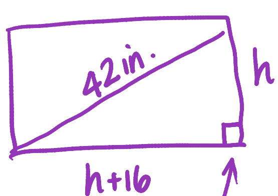
* Check!

➔ When would this method be best?

1) Doesn't Factor

2) Already in the form: $a(x-p)^2 + q = 0$

Ex. 3: A wide-screened television has a diagonal measure of 42 in. The width of the screen is 16 in. more than the height. Determine the dimensions of the screen to the nearest tenth of an inch.




Assume!

$a^2 + b^2 = c^2$

$h^2 + (h+16)^2 = 42^2$

$h^2 + (h+16)(h+16) = 1764$

$h^2 + h^2 + 16h + 16h + 256 = 1764$

GCF=2 $\quad \frac{\cancel{2}h^2}{\cancel{2}} + \frac{32h}{2} - \frac{1508}{2} = \frac{0}{2}$

$h^2 + 16h - 754 = 0$

Graphing or Completing The Square...

$(h^2 + 16h + 64 - 64) - 754 = 0$

$(h^2 + 16h + 64) - 64 - 754 = 0$

$(h+8)^2 - 818 = 0$

$\sqrt{(h+8)^2} = \pm\sqrt{818}$

$h + 8 = \pm\sqrt{818}$

$h = \pm\sqrt{818} - 8$

$h = 20.6, -\cancel{36}.6$ ← height > 0

An **extraneous root** is a solution that does not satisfy any initial restrictions. This often happens in word problems, since we usually need positive solutions.

$\boxed{\text{Dimensions: } 20.6 \text{ in. } 36.6 \text{ in.}}$

## 4.4a: The Quadratic Formula

**Objectives:**
- Solve quadratic equations using the quadratic formula

Warm – up: Simplify the following:

a) $\sqrt{20} = 2\sqrt{5}$

$4 \quad 5$

$2 \quad 2$

b) $5\sqrt{48} = 20\sqrt{3}$

$8 \quad 6$

$2 \quad 4 \quad 2 \quad 3$

$2 \quad 2$

c) $\dfrac{-2-\sqrt{4+24}}{6} = \dfrac{-2-\sqrt{28}}{6}$

$= \dfrac{-2-2\sqrt{7}}{6}$

$= \boxed{\dfrac{-1-\sqrt{7}}{3}}$

28

2 14

2 7

Reduce by 2

Any quadratic equation written in the form $ax^2+bx+c=0$ can be solved using the quadratic formula:

*provided

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

- When would we want to use the formula?
  1) Not factorable
  2) Square not completed
- How could we check our solution(s)?
  Plug them in!

Ex. 1: Use the quadratic formula to solve the following equations.

* Write as $ax^2+bx+c=0$

a) $4x = 5-4x^2$

$4x^2+4x-5=0$

$a=4 \quad b=4 \quad c=-5$

$x = \dfrac{-(4) \pm \sqrt{(4)^2-4(4)(-5)}}{2(4)}$ ← 1st step!

$= \dfrac{-4 \pm \sqrt{96}}{8}$

$\sqrt{96} = 4\sqrt{6}$

$8 \quad 12$

$2 \quad 4 \quad 3 \quad 4$

$2 \quad 2 \quad 2 \quad 2$

$= \dfrac{-4 \pm 4\sqrt{6}}{8}$

$= \boxed{\dfrac{-1 \pm \sqrt{6}}{2}} \cong 0.72, -1.74$

b) $3r^2-4=0$

$a=3 \quad b=0 \quad c=-4$

$x = \dfrac{\cancel{-(0)} \pm \sqrt{(0)^2-4(3)(-4)}}{2(3)}$

$= \dfrac{\pm\sqrt{48}}{6}$

$\sqrt{48} = 4\sqrt{3}$

$8 \quad 6$

$2 \quad 4 \quad 2 \quad 3$

$2 \quad 2$

$= \pm \dfrac{4\sqrt{3}}{6}$

$= \boxed{\pm\dfrac{2\sqrt{3}}{3}} \cong \pm 1.15$

7

# 4.4b: The Discriminant

**Objectives:**

how many solutions?
↙ (2, 1, 0)

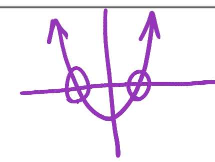
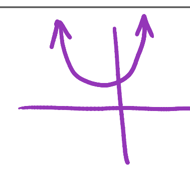
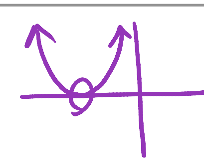
- Use the discriminant to determine the nature of the roots of a quadratic equation.

The Discriminant is everything under the radical sign in the quadratic formula.

$$\boxed{\text{discriminant} = b^2 - 4ac}$$

(+, −)

- The **sign** of the discriminant determines the types of roots of a quadratic equation

| Discriminant | Quadratic Formula | Types of roots (Nature) | Graph |
|---|---|---|---|
| $b^2 - 4ac > 0$ (+) | $x = \frac{-b \pm \sqrt{+}}{2a}$ | 2 solutions |  |
| $b^2 - 4ac < 0$ (−) | $x = \frac{-b \pm \sqrt{-}}{2a}$ undefined! | 0 solutions |  |
| $b^2 - 4ac = 0$ (0) | $x = \frac{-b \pm \sqrt{0}}{2a}$ | 1 solution |  |

Ex. 2: Without solving, determine the nature of the roots of the quadratic equation:

a) $2x^2 + 3x - 10 = 0$

$a = 2 \quad b = 3 \quad c = -10$

$b^2 - 4ac = (3)^2 - 4(2)(-10)$

$= 89 \quad > 0$

→ 2 Solutions!

b) $3x^2 - 7x = -5$

$3x^2 - 7x + 5 = 0$

$a = 3 \quad b = -7 \quad c = 5$

$b^2 - 4ac = (-7)^2 - 4(3)(5)$

$= -11 \quad < 0$

→ 0 Solutions!

Ex. 3: For what values of k does $x^2 + 10x + k = 0$ have 2 equal real roots?

$a=1 \quad b=10 \quad c=k$

→ 1 Solution

$b^2-4ac=0$

$b^2-4ac=0$

$(10)^2-4(1)(k)=0$

$100-4k=0$

$\frac{100}{4}=\frac{\cancel{4}k}{\cancel{4}}$

$\boxed{25=k}$

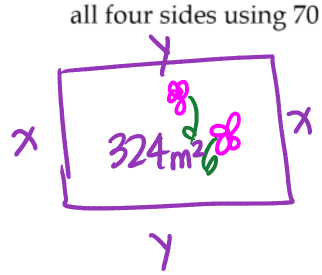
Check: $x^2+10x+25=0$

$(x+5)(x+5)=0$

$x=-5$

Ex. 4: A rectangular garden has an area of 324 square metres. Is it possible to enclose the garden on all four sides using 70 m of fencing? Explain.

→ Is there a solution?




Fence: $\frac{2x+2y=70}{2 \quad 2 \quad 2}$

$x+y=35$

$y=35-x$

Area: $A=xy$

$324=xy$

$324=x(35-x)$

$324=35x-x^2$

$x^2-35x+324=0$

$b^2-4ac=(-35)^2-4(1)(324)$

$=-71 \quad <0$

→ No Solution!

Need more fence!

Name: ____________________________

# Pre-calculus 11

## Chapter 4: QUADRATIC EQUATIONS

**Instructions for Homework**:

- Clearly write your name, class block, and date at the top of your paper.
- Title your page with the textbook section and page #.
- Begin each problem by writing the problem # and copying the question.
- Show all your work clearly. Answers alone will receive no credit.
- Check your own answers in the back of the book with a coloured pen.
- Correct all wrong answers beside your original work. Ask in class or during morning or afternoon extra help times if you need help correcting your own work.

| Topics | Homework | Due Dates | Complete? |
|---|---|---|---|
| 4.1: Graphical solutions of quadratic equations | Pg. 215: # 1 – 4, 6, 9, 10 | Oct. 21/22 | |
| 4.2a: Solving Quadratic Equations by Factoring | Pg. 229: #1 – 4 (a,c only), 7, 10 | Oct. 26/27 | |
| 4.2b: Solving Quadratic Equations by Factoring (harder examples) | Pg. 229: #5, 8, 10, 13, 14 | Oct. 28/29 | |
| 4.3: Solving quad. equations by squaring | Pg. 240: # 4 – 7 (a,c,e only), 8, 10, 14, 15 | Oct. 30/2 | |
| **** QUIZ (4.1, 4.2) ****<br>4.4: The Quadratic Formula | Pg. 254: # 3 – 7, 11 | Nov. 3/4 | |
| 4.4: The Discriminant | Pg. 254: # 1, 2, 8, 12, 21, 23 | Nov. 5/6 | |
| Ch.4 Review | Pg. 261 #: 1 – 14 | Nov. 9/10 | |
| **Ch. 4 TEST** | | Nov. 12/13 | |