Notes

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8:32 AM

5.1: Working with Radicals

→ What is a **radical**?

Index
- tells you what type of root

Radical

$$\sqrt[2]{10}$$

Radicand

Convert Mixed Radicals to Entire Radicals

Ex. 1: Express each mixed radical in entire radical form. Identify the values of the variable for which the radical represents a real number.

a) $7\sqrt[2]{2}$

$= \sqrt{2\cdot 7\cdot 7}$

$= \sqrt{98}$

Entire Radical

b) $a^4\sqrt[2]{a}$

$= \sqrt{a\cdot a^4\cdot a^4}$

$= \sqrt{a^9}$

c) $5b\sqrt[3]{3b^2}$

$= \sqrt[3]{3b^2\cdot 5b\cdot 5b\cdot 5b}$

$= \sqrt[3]{375b^5}$

Radicals in Simplest Form

A radical is in simplest form if the following are true.
- The ***radicand*** does not contain a fraction or any factor that can be removed.
- The radical is not part of the denominator of a fraction.

For example, $\sqrt{18} = \sqrt{3\cdot 3\cdot 2} = 3\sqrt{2}$ ~~$\sqrt{18} = \sqrt{3\cdot 3\cdot 2} = 3\sqrt{2}$~~

Express Entire Radicals as Mixed Radicals

Ex. 2: Convert each entire radical to a mixed radical in simplest form.

a) $\sqrt{200} = 10\sqrt{2}$

200 → 20, 10

20 → 4, 5; 10 → 5, 2

4 → 2, 2

b) $\sqrt[4]{c^9}$

$= \sqrt[4]{c\cdot c\cdot c\cdot c\cdot c\cdot c\cdot c\cdot c\cdot c}$

$= c^2\sqrt[4]{c}$

c) $\sqrt{48y^5}$

48 → 6, 8

6 → 2, 3; 8 → 2, 4

4 → 2, 2

$y\cdot y\cdot y\cdot y\cdot y$

$= 4y^2\sqrt{3y}$

Compare and Order Radicals

Ex. 3: Order the following numbers from least to greatest without a calculator.

$$5,\ 3\sqrt{3},\ 2\sqrt{6},\ \sqrt{23}$$

*Re-write as entire radicals!

$5 = \sqrt{5 \cdot 5} = \sqrt{25}$

$3\sqrt{3} = \sqrt{3 \cdot 3 \cdot 3} = \sqrt{27}$

$2\sqrt{6} = \sqrt{6 \cdot 2 \cdot 2} = \sqrt{24}$

$\sqrt{23}$

$\sqrt{23},\ 2\sqrt{6},\ 5,\ 3\sqrt{3}$

Restrictions on Variables

If a radical represents a real number and has an **even index**, the radicand must be non-negative. $\sqrt{4-x}$

(2, 4, 6, ...) Odd index ok! eg. $\sqrt[3]{-8} = -2$

$$4 - x \geq 0$$
$$4 - x + x \geq 0 + x$$
$$4 \geq x$$

Isolate the variable by applying algebraic operations to both sides of the inequality symbol.

Ex. 4: State the restrictions on the following radical expressions.

a) $\sqrt[2]{x-2} + 1$ Index = 2 (even)

$x - 2 \geq 0$

$\boxed{x \geq 2}$

b) $\sqrt[2]{x-5} + \sqrt[2]{1-x}$ (even)

$x - 5 \geq 0$ $1 - x \geq 0$

$x \geq 5$ $1 \geq x$

Not possible

Like Radicals

Radicals with the same ***radicand*** and ***index*** are called ***like radicals***. When ***adding and subtracting*** radicals, only like radicals can be combined. You may need to convert radicals to a different form (*mixed or entire*) before identifying like radicals.

*Simplify first!

Pairs of Like Radicals	Pairs of Unlike Radicals
$5\sqrt{7}$ and $-\sqrt{7}$	$2\sqrt{5}$ and $2\sqrt{3}$
$\frac{2}{3}\sqrt[3]{5x^2}$ and $\sqrt[3]{5x^2}$	$\sqrt[4]{5a}$ and $\sqrt[5]{5a}$

Add and Subtract Radicals

Ex. 5: Simplify radicals and combine like terms.

a) $\sqrt{50} + 3\sqrt{2}$ $= 5\sqrt{2} + 3\sqrt{2}$

2 25

5 5

$= \boxed{8\sqrt{2}}$

b) $\sqrt{4c} - 4\sqrt{9c}$, $\boxed{c \geq 0}$ restriction

2 2   3 3

$= 2\sqrt{c} - 12\sqrt{c}$

$= \boxed{-10\sqrt{c}}$

c) $-\sqrt{27} + 3\sqrt{5} - \sqrt{80} - 2\sqrt{12}$

9 3   8 10   3 4

3 3   2 4 2 5   2 2

2 2

$= -3\sqrt{3} + 3\sqrt{5} - 4\sqrt{5} - 4\sqrt{3}$

$= \boxed{-7\sqrt{3} - \sqrt{5}}$

5.2: Multiplying and Dividing Radical Expressions

Multiplying Radicals

When multiplying radicals, multiply the coefficients and multiply the radicands. ***You can only multiply radicals if they have the same index.***

$$(2\sqrt{7})(4\sqrt{75}) = (2)(4)\sqrt{(7)(75)}$$
$$= 8\sqrt{525}$$
$$= 5 \cdot 8\sqrt{21}$$
$$= 40\sqrt{21}$$

525

7 75

25 3

5 5

Radicals can be simplified before multiplying:

$$(2\sqrt{7})(4\sqrt{75}) = (2\sqrt{7})\left(4\sqrt{(25)(3)}\right)$$
$$= (2\sqrt{7})[4 \cdot 5\sqrt{3}]$$
$$= (2\sqrt{7})[20\sqrt{3}]$$
$$= 2 \cdot 20\sqrt{7 \cdot 3}$$
$$= 40\sqrt{21}$$

Ex. 1: Multiply, and simplify the products where possible.

a) $(-3\sqrt{2x})(4\sqrt{6})$, $x \geq 0$ Restriction

$= -12\sqrt{12x}$

2 6

2 3

$= \boxed{-24\sqrt{3x}}$

b) $7\sqrt{3}(5\sqrt{5} - 6\sqrt{3})$

$= 35\sqrt{15} - 42\sqrt{9}$

$= 35\sqrt{15} - 42(3)$

$= \boxed{35\sqrt{15} - 126}$

Your Turn

Multiply. Simplify where possible.

a) $5\sqrt{3}(\sqrt{6})$

$= 5\sqrt{18} = \boxed{15\sqrt{2}}$

2 9

3 3

b) $-2\sqrt[3]{11}(4\sqrt[3]{2} - 3\sqrt[3]{3})$

$= \boxed{-8\sqrt[3]{22} + 6\sqrt[3]{33}}$

c) $(4\sqrt{2} + 3)(\sqrt{7} - 5\sqrt{14})$ FOIL

$= 4\sqrt{14} - 20\sqrt{28} + 3\sqrt{7} - 15\sqrt{14}$

2 14

2 7

$= 4\sqrt{14} - 40\sqrt{7} + 3\sqrt{7} - 15\sqrt{14}$

$= \boxed{-11\sqrt{14} - 37\sqrt{7}}$

d) $-2\sqrt{11c}(4\sqrt{2c^3} - 3\sqrt{3}),\ c \geq 0$

$= -8\sqrt{22c^4} + 6\sqrt{33c}$

$= \boxed{-8c^2\sqrt{22} + 6\sqrt{33c}}$

Dividing Radicals

When dividing radicals, ***divide the coefficients*** and then ***divide the radicands***. ***You can only divide radicals that have the same index***.

$$\frac{4\sqrt[3]{6}}{2\sqrt[3]{3}} = 2\sqrt[3]{2}$$

* Not allowed Radicals in the denominator!

Rationalizing Denominators (change the radical denominator to non radical denominator)

To ***simplify*** an expression that has a radical in the denominator, you need to ***rationalize*** the denominator, see the example below.

$$\frac{5}{2\sqrt{3}} = \frac{5}{2\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{5\sqrt{3}}{2(3)} = \boxed{\frac{5\sqrt{3}}{6}} = \frac{~~m~~}{}$$

problem!

For a binomial denominator that contains a square root, multiply both the numerator and denominator by a ***conjugate*** of the denominator.

Conjugate: The same binomial, opposite sign. eg. $x-2$, $x+2$

$$\frac{5\sqrt{3}}{(4-\sqrt{6})}\cdot\frac{(4+\sqrt{6})}{(4+\sqrt{6})} = \frac{20\sqrt{3}+5\sqrt{18}}{16+\cancel{4\sqrt{6}}-\cancel{4\sqrt{6}}-6} = \frac{20\sqrt{3}+5\sqrt{18}}{10}$$

problem!

18 = 2 · 9 = 2 · 3 · 3

$$= \frac{\cancel{20}^{4}\sqrt{3}+\cancel{15}^{3}\sqrt{2}}{\cancel{10}_{2}} = \boxed{\frac{4\sqrt{3}+3\sqrt{2}}{2}}$$

Ex. 2: Simplify each expression.

a) $\frac{\sqrt[3]{5}}{\sqrt[3]{3x}} \times \frac{\sqrt[3]{3x}}{\sqrt[3]{3x}} \times \frac{\sqrt[3]{3x}}{\sqrt[3]{3x}} = \sqrt[3]{27x^3}$

Problem!

$$= \boxed{\frac{\sqrt[3]{45x^2}}{3x}}$$

b) $\frac{11}{(\sqrt{5}+7)}\cdot\frac{(\sqrt{5}-7)}{(\sqrt{5}-7)} = \frac{11\sqrt{5}-77}{5-\cancel{7\sqrt{5}}+\cancel{7\sqrt{5}}-49}$

Problem!

$$= \frac{\cancel{11}^{1}\sqrt{5}-\cancel{77}^{7}}{-\cancel{44}_{4}}$$

$$= \boxed{\frac{\sqrt{5}-7}{-4}} \text{ or } -\frac{(\sqrt{5}-7)}{4} \text{ or } \frac{-\sqrt{5}+7}{4}$$

Your Turn

Simplify each quotient.

a) $\frac{2\sqrt{51}}{\sqrt{3}} = \boxed{2\sqrt{17}}$

Divide!

b) $\frac{6}{\sqrt{4x+1}}, x>0$

$$\frac{6}{(2\sqrt{x}+1)} \times \frac{(2\sqrt{x}-1)}{(2\sqrt{x}-1)} = \frac{12\sqrt{x}-6}{4x-\cancel{2\sqrt{x}}+\cancel{2\sqrt{x}}-1}$$

$$= \boxed{\frac{12\sqrt{x}-6}{4x-1}}$$

5.3: Radical Equations (day 1)

➔ What is a *radical equation?*

Not: $\sqrt{2} = x$

$(\sqrt{x})^2 = (4)^2 \rightarrow x = 16$

Solving a radical equation is similar to solving a linear or quadratic equation, we are trying to **isolate the variable** and determine possible values for the unknown.

Ex. 1: **a)** State the ***restrictions*** on x in $5+\sqrt{2x+1}=12$ if the radical is a real number.

$2x+1 \geq 0$

$2x \geq -1$

$x \geq -\frac{1}{2}$

b) Solve $5+\sqrt{2x+1}=12$

$(\sqrt{2x+1})^2 = (7)^2$

$2x+1 = 49$

$2x = 48$

$\boxed{x = 24}$

Check:

$5+\sqrt{2(24)+1} = 12$

$5+\sqrt{49} = 12$ ✓

Your Turn

Identify any restrictions on y in $-2+\sqrt{y-1}=3$ if the radical is a real number. Then, solve the equation.

Restriction: $y-1 \geq 0$

$y \geq 1$

Solve: $-2+\sqrt{y-1} = 3$

$(\sqrt{y-1})^2 = (5)^2$

$y-1 = 25$

$\boxed{y = 26}$

Check:

$-2+\sqrt{26-1} = 3$

$-2+\sqrt{25} = 3$ ✓

Ex. 2: Identify the restrictions on n in $n-\sqrt{5-n}=-7$ if the radical is a real number. Then, solve the equation.

Restriction: $5-n \geq 0$

$5 \geq n$

Solve: $n-\sqrt{5-n}=-7$

$-\sqrt{5-n}=-7-n$

$(\sqrt{5-n})^2=(7+n)^2$

FOIL!

$5-n=(7+n)(7+n)$

$5-n=49+7n+7n+n^2$

$0=n^2+15n+44$

$0=(n+11)(n+4)$

$n=-11,-4$

extraneous root

Quadratic!
- Set = 0
- Factor

Check: $-11-\sqrt{5-(-11)}=-7$ ✗

$-4-\sqrt{5-(-4)}=-7$ ✓

$\boxed{\text{only } x=-4}$

To solve radical equations:

Optional

1. State any **restrictions** on the variables.
2. Isolate the radical. Square both sides.
3. Solve the remaining quadratic equation.
4. Check your solution(s). Reject any **extraneous roots**.

Extraneous roots are solutions that do not satisfy any initial conditions.

Your Turn

Identify any restrictions on m in $m-\sqrt{2m+3}=6$ if the radical is a real number. Then, solve the equation. Check your solution(s).

Restrictions: $2m+3 \geq 0$

$m \geq -3/2$

Solve: $m-\sqrt{2m+3}=6$

$(-\sqrt{2m+3})^2=(6-m)^2$

$2m+3=(6-m)(6-m)$

$2m+3=36-6m-6m+m^2$

$0=m^2-14m+33$

$0=(m-11)(m-3)$

$m=11,3$

Check:

✓ $11-\sqrt{2(11)+3}=6$

✗ $3-\sqrt{2(3)+3}=6$

$\boxed{\text{only } m=11}$

5.3: Radical Equations (day 2)

Objectives:

- Solving equations involving 2 square roots
- Applications of radical equations

Ex.1: Solve $7+\sqrt{3x}=\sqrt{5x+4}+5$

$(\sqrt{3x})^2=(\sqrt{5x+4}-2)^2$

$3x=(\sqrt{5x+4}-2)(\sqrt{5x+4}-2)$

$3x=5x+4-2\sqrt{5x+4}-2\sqrt{5x+4}+4$

$3x=5x+8-4\sqrt{5x+4}$

$-2x-8=-4\sqrt{5x+4}$

$(2x+8)^2=(4\sqrt{5x+4})^2$

$(2x+8)(2x+8)=16(5x+4)$

⋮

- Set $=0$
- Factor
- Check!

$\boxed{x=0,12}$

Your Turn: Solve $\sqrt{3+x}+\sqrt{2x-1}=5$

$(\sqrt{3+x})^2=(5-\sqrt{2x-1})^2$

$3+x=(5-\sqrt{2x-1})(5-\sqrt{2x-1})$

$3+x=25-5\sqrt{2x-1}-5\sqrt{2x-1}+2x-1$

$3+x=24+2x-10\sqrt{2x-1}$

$(-21-x)^2=(-10\sqrt{2x-1})^2$

$(-21-x)(-21-x)=100(2x-1)$

$441+21x+21x+x^2=200x-100$

$x^2-158x+541=0$

⋮

Ew...
Quad. Formula
or Graph. Calc.

Ex. 2: What is the speed, in metres per second, of a 0.4 kg football that has 28.8 kJ of Kinetic energy? Use the kinetic energy formula, $E_K=\frac{1}{2}mv^2$, where E_K represents the kinetic energy, in joules; m represents the mass, in kilograms; and v represents the speed, in metres per second.