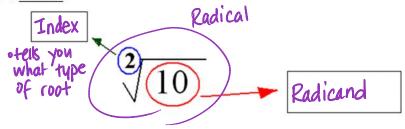
Notes

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5.1: Working with Radicals

→ What is a radical?



Convert Mixed Radicals to Entire Radicals

Ex. 1: Express each mixed radical in entire radical form. Identify the values of the variable for which the radical represents a real number.

a)
$$7\sqrt[3]{2}$$

b) $a^{3}\sqrt[3]{a}$
 $=\sqrt{2\cdot7\cdot7}$
 $=\sqrt{a\cdot a^{4}\cdot a^{4}}$
 $=\sqrt[3]{3b^{2}\cdot5b\cdot5b\cdot5b}$
 $=\sqrt{98}$
 $=\sqrt{a^{9}}$
 $=\sqrt{375b^{5}}$

(Entire Padical

Radicals in Simplest Form

A radical is in simplest form if the following are true.

- The *radicand* does not contain a <u>fraction</u> or any factor that can be removed.
- The radical is not part of the denominator of a fraction.

For example,
$$\sqrt{18} = \sqrt{3 \cdot 3}/2 = 3\sqrt{2}$$

Express Entire Radicals as Mixed Radicals

Ex. 2: Convert each entire radical to a mixed radical in simplest form.

a)
$$\sqrt{200} = 10\sqrt{2}$$

b) $\sqrt{48y^5}$
 $20\ 10$
 $4\ 5\ 3)\ 2$
 $= c^2 4c$
 $= 4y^2 \sqrt{3}y$
 $= 4y^2 \sqrt{3}y$

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Compare and Order Radicals

Ex. 3: Order the following numbers from least to greatest without a calculator.

12. 3. Order the following numbers from least to greatest without a
$$2.5$$
, $3\sqrt{3}$, $2\sqrt{6}$, $\sqrt{23}$ entire radicals! $5 = \sqrt{5.5} = \sqrt{25}$ $3\sqrt{3} = \sqrt{3.3.3} = \sqrt{27}$ $2\sqrt{6} = \sqrt{6.2.2} = \sqrt{24}$ $\sqrt{23}$, $2\sqrt{6}$, 5 , $3\sqrt{3}$ $\sqrt{23}$

Restrictions on Variables

If a radical represents a real number and has an <u>even index</u>, the radicand must be non-negative. (2,4,6,...) Odd index ok! eq.

$$4 - x \ge 0$$

$$4 - x + x \ge 0 + x$$

$$4 \ge x$$

Isolate the variable by applying algebraic operations to both sides of the inequality symbol.

Ex. 4: State the restrictions on the following radical expressions.

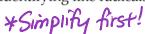
a)
$$\sqrt[2]{x-2}+1$$
 Index = 2 (even)

$$x-2 \ge 0$$

$$x \ge 5$$

Like Radicals

Radicals with the same <u>radicand</u> and <u>index</u> are called *like radicals*. When <u>adding and subtracting</u> radicals, only like radicals can be combined. You may need to convert radicals to a different form (<u>mixed or entire</u>) before identifying like radicals.



Pairs of Like Radicals	Pairs of Unlike Radicals
$5\sqrt{7}$ and $-\sqrt{7}$	$2\sqrt{5}$ and $2\sqrt{3}$
$\frac{2}{3}\sqrt[3]{5x^2}$ and $\sqrt[3]{5x^2}$	∜ <u>5a</u> and ∜ <u>5a</u>

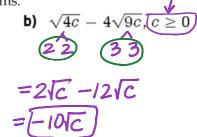
restriction

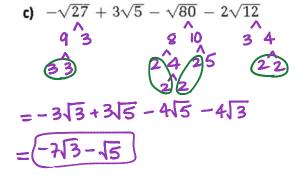
Add and Subtract Radicals

Ex. 5: Simplify radicals and combine like terms.

a)
$$\sqrt{50} + 3\sqrt{2} = 5\sqrt{2} + 3\sqrt{2}$$

2 25 = $8\sqrt{2}$





5.2: Multiplying and Dividing Radical Expressions

Multiplying Radicals

When multiplying radicals, multiply the coefficients and multiply the radicands. <u>You can</u> only multiply radicals if they have the same index.

Radicals can be simplified before multiplying:

$$(2\sqrt{7})(4\sqrt{75}) = (2)(4)\sqrt{(7)(75)}$$

 $= 8\sqrt{525}$
 $= 5 \cdot 8\sqrt{21}$
 $= 40\sqrt{21}$
Radicals can be simplified before multiplying:
 $(2\sqrt{7})(4\sqrt{75}) = (2\sqrt{7})(4\sqrt{(25)(3)})$
 $= (2\sqrt{7})[4 \cdot 5\sqrt{3}]$
 $= (2\sqrt{7})[20\sqrt{3}]$
 $= 2 \cdot 20\sqrt{7 \cdot 3}$
 $= 40\sqrt{21}$

Ex. 1: Multiply, and simplify the products where possible.

a)
$$(-3\sqrt{2x})(4\sqrt{6}), x \ge 0$$

b) $7\sqrt{3}(5\sqrt{5} - 6\sqrt{3})$
 $= -12\sqrt{12x}$
 $= 35\sqrt{15} - 42\sqrt{9}$
 $= 35\sqrt{15} - 42(3)$
 $= -24\sqrt{3}x$

Your Turn

Multiply. Simplify where possible.

a)
$$5\sqrt{3}(\sqrt{6})$$

= $5\sqrt{18}$ = $15\sqrt{2}$ = $-8\sqrt[3]{22} + 6\sqrt[3]{3}$
c) $(4\sqrt{2} + 3)(\sqrt{7} - 5\sqrt{14})$ Foll d) $-2\sqrt{11c}(4\sqrt{2c^3} - 3\sqrt{3}), c \ge 0$
= $4\sqrt{14} - 20\sqrt{28} + 3\sqrt{7} - 15\sqrt{14}$ = $-8\sqrt{22}c^4 + 6\sqrt{33}c$
= $4\sqrt{14} - 40\sqrt{7} + 3\sqrt{7} - 15\sqrt{14}$ = $-8c^2\sqrt{22} + 6\sqrt{33}c$
= $-1\sqrt{14} - 37\sqrt{7}$

Dividing Radicals

When dividing radicals, divide the coefficients and then divide the radicands. You can only divide radicals that have the same index.

$$\left(\frac{4\sqrt[3]{6}}{2\sqrt[5]{3}}\right) = 2\sqrt[3]{2}$$

* Not allowed Radicals in the denominator!

Rationalizing Denominators (change the radical denominator to non radical denominator)

To <u>simplify</u> an expression that has a radical in the denominator, you need to <u>rationalize</u> the denominator, see the example below.

$$\frac{5}{2\sqrt{3}} = \frac{5}{2\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{5\sqrt{3}}{2(3)} = \frac{5\sqrt{3}}{2}$$
problem!

For a binomial denominator that contains a square root, multiply both the numerator and denominator by a *conjugate* of the denominator.

Conjugate: The same binomial, opposite sign. eg. x-2, x+2

$$\frac{5\sqrt{3}}{(4+\sqrt{6})} \frac{(4+\sqrt{6})}{(4-\sqrt{6})} = \frac{20\sqrt{3}+5\sqrt{18}}{16+4\sqrt{6}-4\sqrt{6}-6} = \frac{20\sqrt{3}+5\sqrt{8}}{10} = \frac{2\sqrt{3}+2\sqrt{2}}{2\sqrt{9}} = \frac{4\sqrt{3}+3\sqrt{2}}{2\sqrt{9}} = \frac{4\sqrt{3}+3\sqrt{2}}{2\sqrt{9}}$$

Ex. 2: Simplify each expression.

a)
$$\frac{\sqrt[3]{5}}{\sqrt[3]{3}} \times \frac{\sqrt[3]{3}}{\sqrt[3]{3}} \times \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \sqrt[3]{27} \times \frac{11}{\sqrt[3]{5} + 7} = \frac{11 \times 5 - 77}{5 - 215 + 315 - 49}$$

Problem! = $\frac{\sqrt[3]{45} \times 2}{3 \times 3}$

Your Turn

Simplify each quotient.

a)
$$\frac{2\sqrt{51}}{\sqrt{3}} = \boxed{2\sqrt{17}}$$
Divide!

b)
$$\frac{6}{\sqrt{4x+1}}, x>0$$

$$\frac{6}{\sqrt{4x+1}}, \frac{(2\sqrt{x}-1)}{4} = \frac{12\sqrt{x}-6}{4x-2\sqrt{x}+2\sqrt{x}-1}$$

$$= \frac{12\sqrt{x}-6}{4x-1}$$

5.3: Radical Equations (day 1)

What is a radical equation? Not:
$$\sqrt{2} = \chi$$

$$(\sqrt{\chi})^2 = (4)^2 \Rightarrow \chi = 16$$

Solving a radical equation is similar to solving a linear or quadratic equation, we are trying to **isolate the variable** and determine possible values for the unknown.

Ex. 1: a) State the <u>restrictions</u> on x in $5+\sqrt{2x+1}=12$ if the radical is a real number.

b) Solve
$$(3+\sqrt{2x+1}=12)$$
 $(\sqrt{2x+1})=(7)^2$
 $2x+1=49$
 $2x=48$
 $x \ge -1$
 $5+\sqrt{2(24)+1}=12$
 $2x=48$
 $5+\sqrt{49}=12$
 1

Your Turn

Identify any restrictions on y in $-2 + \sqrt{y-1} = 3$ if the radical is a real number. Then, solve the equation.

Restriction:
$$y-1 \ge 0$$

 $y \ge 1$
Solve: $-2 + \sqrt{y-1} = 3$ Check:
 $(\sqrt{y-1})^2 = (5)^2$ $-2 + \sqrt{26-1} = 3$
 $y-1 = 25$ $-2 + \sqrt{25} = 3$ V
 $y=26$

Identify the restrictions on *n* in $n - \sqrt{5 - n} = -7$ if the radical is a real number. Then, Ex. 2:

Solve the equation.
Restriction:
$$5-n \ge 0$$

 $5 \ge n$
Solve: $n - \sqrt{5-n} = -7$
 $-\sqrt{5-n} = -7 - n$
 $(\sqrt{5-n})^2 = (7+n)^2$
Foll! Check: $-11 - \sqrt{5-(-1)} = -7$ X

Check: -11- (5-(-11) = -7 X

To solve radical equations:

- Optional
- 1. State any **restrictions** on the variables.
- 2. Isolate the radical. Square both sides.
- 3. Solve the remaining quadratic equation.
- ★ 4. Check your solution(s). Reject any extraneous roots.

Extraneous roots are solutions that do not satisfy any initial conditions.

Your Turn

Identify any restrictions on m in $m - \sqrt{2m+3} = 6$ if the radical is a real number. Then, solve the equation. Check your solution(s).

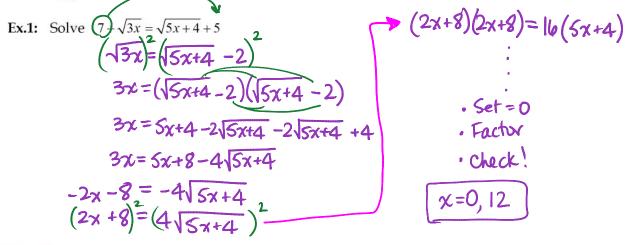
Solve:
$$m - \sqrt{2m+3} = 6$$

 $(-\sqrt{2m+3})^2 = (6-m)^2$
 $2m+3 = (6-m)(6-m)$
 $2m+3 = 36-6m-6m+m^2$
 $0 = m^2 - 14m + 33$
 $0 = (m-11)(m-3)$
 $m = 11, 3$
Check:
 $11 - \sqrt{2(11)+3} = 6$
 $\times 3 - \sqrt{2(3)+3} = 6$

5.3: Radical Equations (day 2)

Objectives:

- Solving equations involving 2 square roots
- · Applications of radical equations



Your Turn: Solve
$$\sqrt{3+x} + \sqrt{2x-1} = 5$$

$$(\sqrt{3+x})^{2} = (5-\sqrt{2x-1})^{2}$$

$$3+x = (5-\sqrt{2x-1})(5-\sqrt{2x-1})$$

$$3+x = 25-5\sqrt{2x-1}-5\sqrt{2x-1}+2x-1$$

$$3+x = 24+2x-10\sqrt{2x-1}$$

$$(-21-x)^{2} = (-10\sqrt{2x-1})^{2}$$

$$(-21-x)(-21-x) = |00(2x-1)|$$

$$(2x-1)^{2} = (-10\sqrt{2x-1})$$

$$(-21-x)(-21-x) = |00(2x-1)|$$

Ex. 2: What is the speed, in metres per second, of a 0.4 kg football that has 28.8 kJ of Kinetic energy? Use the kinetic energy formula, $E_K = \frac{1}{2}mv^2$, where E_K represents the kinetic energy, in joules; m represents the mass, in kilograms; and v represents the speed, in metres per second.