Notes

December-02-15
8:32 AM

9.1: Linear Inequalities in Two Variables

Review:

1. Write an inequality for each graph:

a) not included

-8 -7 -6 -5 -4

$x > -6$

b) included

0 1 2 3 4

$0 \le x \le 4$

2. Show the given inequality on a number line:

a) $-2 < x \le 5$

-2 5

b) $x \neq 2, x < 4$

2 4

3. Solve each of the following inequalities. Graph the solution on a number line.

a) $2(3-x)-1 \ge 7$

$6-2x-1 \ge 7$

$-2x+5 \ge 7$

$\frac{\cancel{-2}x}{\cancel{-2}} \ge \frac{2}{-2}$ ← FLIP sign!

$\boxed{x \le -1}$

Check!

b) $\left(\frac{3x}{4} + \frac{x}{2} > 5\right) \times 4$

$3x + 2x > 20$

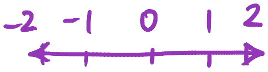
$\frac{\cancel{5}x}{\cancel{5}} > \frac{20}{5}$

$\boxed{x > 4}$

Solving inequalities follows the same process as equations. The only exception to the rule is if:

Multiply/Divide by a negative -> FLIP inequality sign!

Why?



-2 -1 0 1 2

$\frac{-2}{-1} < \frac{-1}{-1}$ $1 < 2$

$2 > 1$

Linear Inequalities in 2 variables:

Consider $y \ge 2x+1$

$y > 2x+1$ OR $y = 2x+1$ line!

- We will graph what we'll call the **boundary line** of this equation ($y = 2x+1$) and shade either the region above (if y > line) or below (if y < line)
- The **solution region** will be all the shaded area, including the boundary, if appropriate.

- We use a **SOLID** boundary line if we have $\leq$ or $\geq$ (line included)
- We use a **DOTTED** boundary line if we have $<$ or $>$ (line not included)

"Solution Region"

Ex. 1: Graph $y \geq 2x + 1$

Graph Boundary: $y = 2x + 1$ (solid)

slope y-int

$y \geq$ line → "greater than"

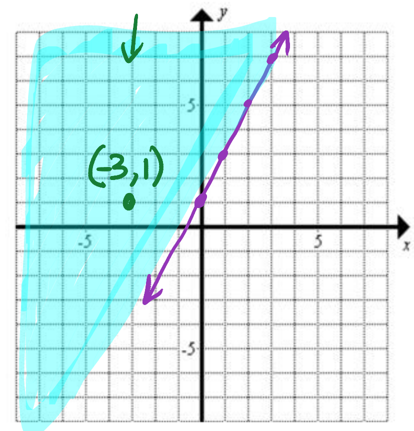
→ shade above

Check: (−3, 1)

$1 \geq 2(-3) + 1$

$1 \geq -6 + 1$

$1 \geq -5$ ✓



Ex. 2: Graph $4x - 3y < -12$

$\frac{4x}{3} + \frac{12}{3} < \frac{3y}{3}$

$\frac{4}{3}x + 4 < y \rightarrow y > \frac{4}{3}x + 4$

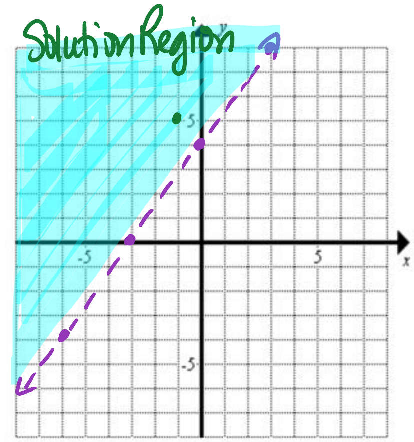
Graph Boundary

$y = \frac{4}{3}x + 4$

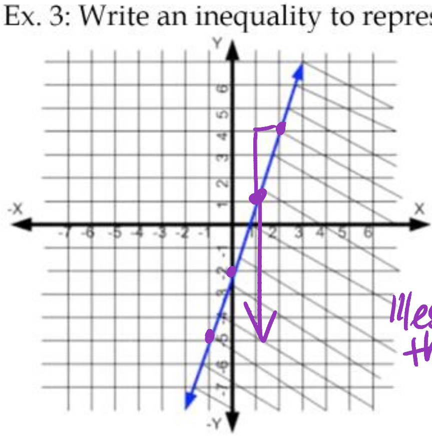
(dotted)

$y >$ line "greater than"

Solution Region



Ex. 3: Write an inequality to represent the graph.



"less than"

Boundary: $y = mx + b$

$y = 3x - 2$ (solid)

$\boxed{y \leq 3x - 2}$

Ex. 4: At a garden store a flat of marigolds costs \$5 each and a flat of petunias costs \$6 each, tax included. You have a total of \$60 to spend. let m= marigolds , p= petunias

a) Write an inequality to represent the flowers you could purchase.

$5m + 6p \leq 60$

b) What are the restrictions on the variables, if any?

max (60/5) max (60/6)

$0 \leq m \leq 12$ $0 \leq p \leq 10$

↑intercept!

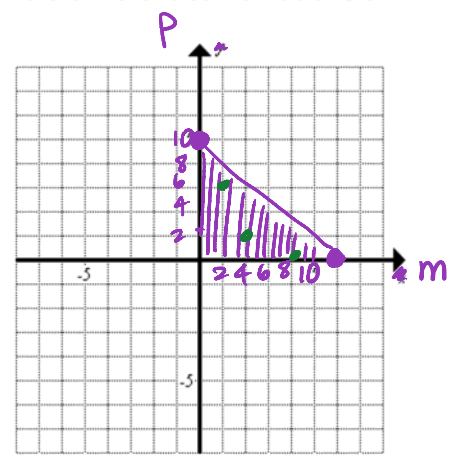
c) Graph the inequality, and use the graph to find 3 different combinations of purchases that you could make.

$5m + 6p \leq 60$

→ Rearranging not nice

→ use restrictions to graph!

Can afford any point in shaded region



9.2: Quadratic Inequalities in One Variable

→ only x

We will now be graphing and solving **quadratic inequalities.** The graphs will no longer be linear, but will have a parabolic shape.

Consider $x^2 - 4x - 5 \geq 0$

We already know how to solve $x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$\boxed{x = 5, -1}$

What about $x^2 - 4x - 5 > 0$?

As in 9.1, we will shade either above or below, but instead of above or below the line (y), it will be along the **x-axis (0)**, corresponding to where the graph is above or below.

This time, we will state our solution as an inequality.

Method 1: Graphically

Ex. 1: Solve $x^2 - 4x - 5 \geq 0$

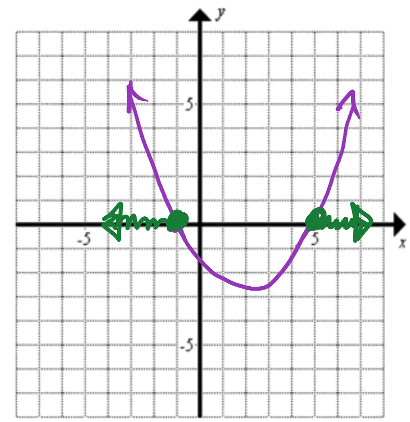
① Find zeros: $x^2 - 4x - 5 = 0$

$x = 5, -1$

② Sketch (approx.) $y = x^2 - 4x - 5$

→ opens up

→ "greater than"



$\boxed{\begin{array}{l} x \leq -1 \\ x \geq 5 \end{array}}$

Method 2: Roots and Test Points

Ex. 2: Solve $x^2 + 5x - 6 < 0$

① Find zeros: $x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

$x = -6, 1$

② -6, 1

$x = -11$	$x = -1$	$x = 2$
$(-11)^2 + 5(-11) - 6$	$(-1)^2 + 5(-1) - 6$	$(2)^2 + 5(2) - 6$
$60 < 0$	$-10 < 0$	$8 < 0$
X	✓	X

$\boxed{-6 < x < 1}$

$x > -6$

$x < 1$

Method 3: Sign Analysis

Ex. 3: $x^2 - 2x - 3 > 0$

① Find zeros: $x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

② -1, 3

$x = -2$	$x = 0$	$x = 1000000$
$(-)(-)$	$(-)(+)$	$(+)(+)$
$+ > 0$	$- > 0$	$+ > 0$
✓	X	✓

$\boxed{\begin{array}{l} x < -1 \\ x > 3 \end{array}}$

*Sign Analysis only works in factored form!

YOUR TURN

Solve $-x^2+x+12<0$ using any of the 3 methods shown above.

$0<x^2-x-12$ → "greater than"

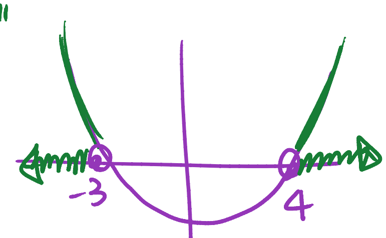
① Find zeros:

$0=x^2-x-12$

$0=(x-4)(x+3)$

$x=4,-3$

②



$\boxed{\begin{matrix}x<-3\\ x>4\end{matrix}}$

Ex. 4: Solve $x^2-4x>10$ using any method.

$x^2-4x-10>0$

① $x^2-4x-10=0$ ← Not factorable...

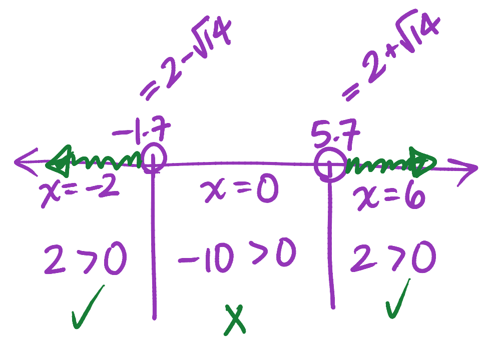
$x=\dfrac{-(-4)\pm\sqrt{(-4)^2-4(1)(-10)}}{2(1)}$

$=\dfrac{4\pm\sqrt{56}}{2}$

56
2 28
2 14
2 7

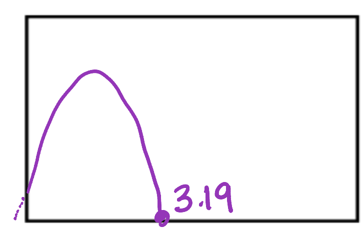
$=\dfrac{4\pm2\sqrt{14}}{2}=2\pm\sqrt{14}\cong 5.7,\ -1.7$

②



$\boxed{\begin{matrix}x<2-\sqrt{14}\\ x>2+\sqrt{14}\end{matrix}}$

Ex. 5: If a baseball is thrown at an initial speed of 15 m/s from a height of 2 m above the ground, the inequality $-4.9t^2+15t+2>0$ models the time, t, in seconds, that the baseball is in flight. During what time interval is the baseball in flight?



$Y_1=-4.9x^2+15x+2$

x [0, 10] y [0, 20]

$\boxed{0<t<3.19}$

9.3: Quadratic Inequalities in Two Variables

Ex. 1: Determine if the point (2,1) is a solution to the inequality $y > (x-4)^2 - 2$

$1 > (2-4)^2 - 2$

$1 > (-2)^2 - 2$

$1 > 2$ X No – not a solution!

As in 9.1, we are now comparing to the function y, i.e. the graph itself. To solve a quadratic inequality in two variables we will:

1. Graph the parabola (boundary). Use a **solid** line if ≤ or ≥ and a **dotted** line if < or >.
2. Determine if we are shading above (y >) or below (y <)
3. Our solution will again be shown graphically.
   → "solution region"

Ex. 2: Solve $y \geq x^2 - 2x - 3$.

① Graph Boundary: (solid)

$y = x^2 - 2x - 3$

$y = (x-3)(x+1)$

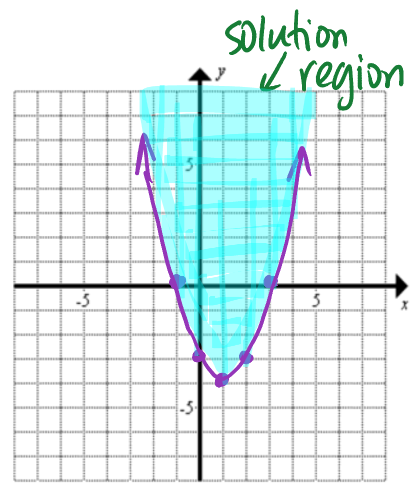
$x = 3, -1$

vertex: $(1, -4)$

② $y \geq x^2 - 2x - 3$

"greater-than"

→ shade above



Ex. 3: Solve $y < -2(x-3)^2 + 1$. Use your graph to determine if (2, -4) is a solution.

① Graph (dotted) $y = -2(x-3)^2 + 1$

vertex: $(3, 1)$

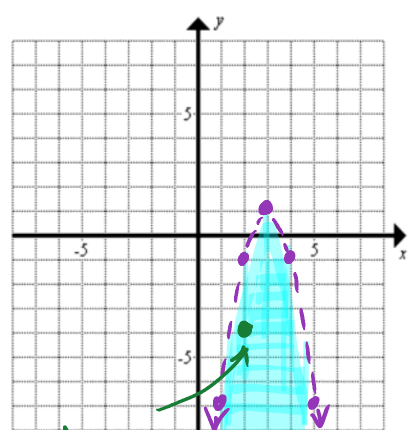
stretch: -2

over	down
1	-2
2	-8

② $y < -2(x-3)^2 + 1$

→ "less than"

→ shade below



(2,-4) is a solution

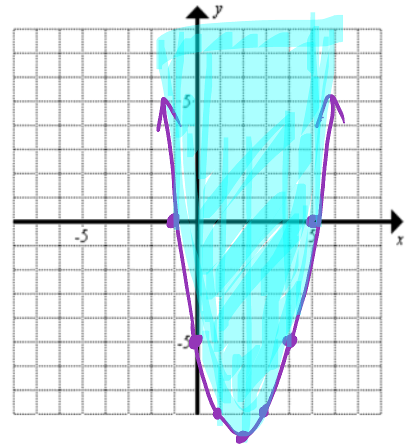
6

YOUR TURN

Solve $y \ge x^2 - 4x - 5$

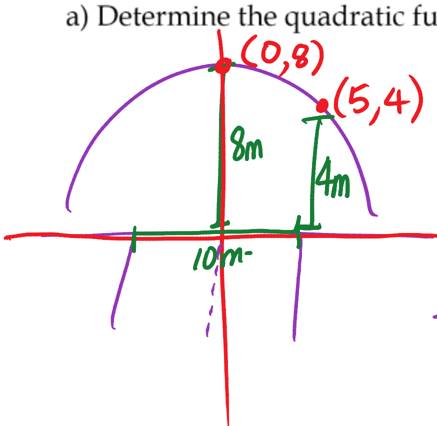
① Graph: $y = x^2 - 4x - 5$
(solid) $= (x-5)(x+1)$
$x = 5, -1$
$y = (2)^2 - 4(2) - 5$
$= -9 \rightarrow (2, -9)$

② "greater than"



Ex. 4: Highway 1 goes through the Cassiar tunnel on the South side of the North Shore. The highest point of the tunnel is 8 m high. The road is 10 m side, and the minimum height of the tunnel above the road is 4 m high.
wide

a) Determine the quadratic function that models the parabolic arch of the tunnel.

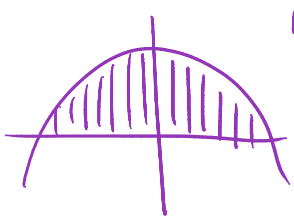


$y = a(x-p)^2 + q$
$y = a(x-0)^2 + 8$
$4 = a(5-0)^2 + 8$
$4 = 25a + 8$
$-4 = 25a$
$a = -4/25$

$\boxed{y = -\frac{4}{25}x^2 + 8}$

b) What is the inequality that represents the space under the tunnel in quadrants I and II?

(y≥0)
II | I



"less than"

$\boxed{\begin{array}{l} y < -\frac{4}{25}x^2 + 8 \\ y \ge 0 \end{array}}$