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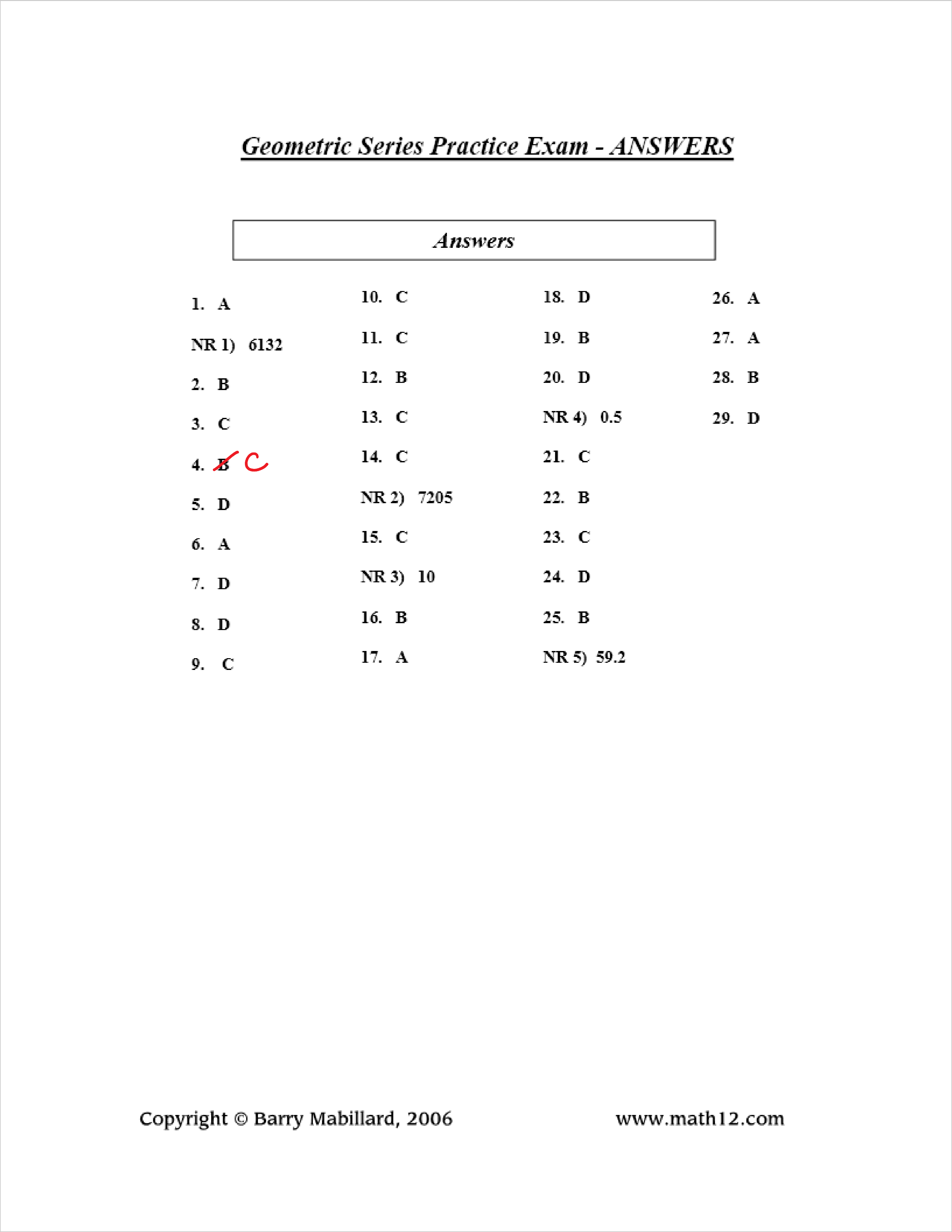

Geometric Series Practice Exam - ANSWERS

Answers

1. A

NR 1) 6132

2. B

3. C

4. ~~B~~ C

5. D

6. A

7. D

8. D

9. C

10. C

11. C

12. B

13. C

14. C

NR 2) 7205

15. C

NR 3) 10

16. B

17. A

18. D

19. B

20. D

NR 4) 0.5

21. C

22. B

23. C

24. D

25. B

NR 5) 59.2

26. A

27. A

28. B

29. D

1. To figure this question out, expand each possible answer to see what the first few terms are:

$\sum_{n=1}^{k}\left(\sqrt{2}\right)^{n}=\sqrt{2}+\left(\sqrt{2}\right)^{2}+\left(\sqrt{2}\right)^{3}+\ldots$ **Common ratio =** $\sqrt{2}$

$\sum_{n=1}^{k}\left(2+\sqrt{2}\right)^{n}=\left(2+\sqrt{2}\right)+\left(2+\sqrt{2}\right)^{2}+\left(2+\sqrt{2}\right)^{3}+\ldots$ **Common ratio =** $\left(2+\sqrt{2}\right)$

$\sum_{n=1}^{k}\sqrt{2}n=\sqrt{2}(1)+\sqrt{2}(2)+\sqrt{2}(3)+\ldots$ **No common ratio**

$\sum_{n=1}^{k}\left(2+\sqrt{2}\right)n=\left(2+\sqrt{2}\right)+\left(2+\sqrt{2}\right)(2)+\left(2+\sqrt{2}\right)(3)+\ldots$ **No common ratio**

The answer is **A**.

NR 1) $\sum_{k=1}^{9}6(2)^{k}=12+24+\ldots$

$a=12 \quad r=2$

$$S_9=\frac{a\left(r^{n}-1\right)}{r-1}$$

$$S_9=\frac{12\left(2^{9}-1\right)}{2-1}$$

$S_9=6132$

The answer is **6132**.

2.

$\log_3 3+\log_3 9+\log_3 27+\log_3 81$

$\log_3 3+\log_3 3^2+\log_3 3^3+\log_3 3^4$

$\log_3 3+2\log_3 3+3\log_3 3+4\log_3 3$

$1+2+3+4$

$=\sum_{n=1}^{4}n$

The answer is **B**.

3. $\frac{1}{3}+\frac{4}{3}+\frac{16}{3}+\ldots\ldots\frac{4096}{3}$

First determine how many terms are in the series:

$t_n=ar^{n-1}$

$\frac{4096}{3}=\frac{1}{3}(4)^{n-1}$

$4096=(4)^{n-1}$

$4^6=(4)^{n-1}$

$6=n-1$

$n=7$

Then evaluate the sum:

$$S_n=\frac{a\left(1-r^{n}\right)}{1-r}$$

$$S_7=\frac{\frac{1}{3}\left(1-4^{7}\right)}{1-4}$$

$$S_7=\frac{\frac{1}{3}\left(1-4^{7}\right)}{1-4}$$

$S_7=1820.33$

The answer is **C**.

Alternative Solution:

$$S_n=\left(\frac{rt_n-a}{r-1}\right)$$

$$S_n=\left(\frac{4\left(\frac{4096}{3}\right)-\frac{1}{3}}{4-1}\right)$$

$$S_n=\left(\frac{4\left(\frac{4096}{3}\right)-\frac{1}{3}}{4-1}\right)$$

$S_n=1820.33$

4. The number of terms is (Top – Bottom) +1

(41 – 21) + 1 = 21
The answer is **B**.

5. Use the sum formula $S_n = \frac{a\left(r^n - 1\right)}{r-1}$, where $S_n = 5.25$, $n = 6$, and $r = -0.5$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$
$$5.25 = \frac{a\left(1-(-0.5)^6\right)}{1-(-0.5)}$$
$$5.25 = 0.65625a$$
$$a = 8$$

The answer is **D**.

6. Use the following setup

$$\_\_, \_\_, \_\_, \frac{4}{3}, \_\_, \_\_, \frac{32}{81}$$

Pretend this is the first term.
$a = \frac{4}{3}$

Then this is the fourth term.
$t_4 = \frac{32}{81}$, $n = 4$

$$t_n = ar^{n-1}$$
$$\frac{32}{81} = \frac{4}{3}r^{4-1}$$
$$\frac{32}{81} = \frac{4}{3}r^3$$
$$\frac{8}{27} = r^3 \quad \left(\frac{32}{81} \div \frac{4}{3} = \frac{32}{81} \times \frac{3}{4} = \frac{8}{27}\right)$$
$$r = \frac{2}{3}$$

The answer is **A**.

7. Use $t_n = ar^{n-1}$ to determine the number of swings.

$$21 = 40\left(\frac{15}{16}\right)^{n-1}$$
$$0.525 = (0.9375)^{n-1}$$

Solve by graphing
$n = 11$

The answer is **D**.

8.

$$S_n = \frac{a}{1-r}$$
$$S_\infty = \frac{40}{1-\frac{15}{16}}$$
$$S_\infty = 640 \text{ m}$$

The answer is **D**.

9. The area of the original is 144 cm^2

The area after 13 photocopies is the 14th term.

144 cm^2 | ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___ | *ANSWER*

(Vertical lines represent one photocopy)

$$t_{14} = 144(0.8)^{n-1}$$
$$t_{14} = 144(0.8)^{14-1}$$
$$t_{14} = 7.92\ \text{cm}^2$$

The answer is **C**.

10. Since we want to determine the **<u>remaining</u>** mass of impurities, use 0.28 for r.

The amount remaining after seven filters will be the EIGHTH term.

10 g | ___ | ___ | ___ | ___ | ___ | ___ | *ANSWER* *(Vertical lines represent filters)*

$$t_8 = 10(0.28)^{8-1}$$
$$t_8 = 10(0.28)^7$$
$$t_8 = 0.0014\ \text{g}$$

The answer is **C**.

11. Use the following setup:

__, __ ,__, 48, __, 192

Pretend this
is the first term.
$a = 48$

Then this is the
third term.
$t_3 = 192$

$$t_n = ar^{n-1}$$
$$192 = 48r^{3-1}$$
$$192 = 48r^2$$
$$4 = r^2$$
$$r = 2$$

Using this r value, find
the preceding terms.

6, 12 , 24, 48, __, 192

$a = 6$

Now find the sum using

$$S_n = \frac{a(1-r^n)}{1-r}$$
$$S_9 = \frac{6(1-2^9)}{1-2}$$
$$S_9 = 3066$$

The answer is **C**.

12.

$$\sum_{k=3}^{6} \log_k k^3$$

$$= \sum_{k=3}^{6} 3\log_k k$$

$$= 3\log_3 3 + 3\log_4 4 + 3\log_5 5 + 3\log_6 6$$

$$= 3(1) + 3(1) + 3(1) + 3(1)$$

$$= 12$$

The answer is **B**.

13. The only geometric sequence is answer **C**, since a common ratio of $\sqrt{3}$ exists.

The answer is **C**.

14. Write out the first few terms

15 + 45 + …

$a = 15$, $r = 3$, $n = 10$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$S_{10} = \frac{15\left(1-3^{10}\right)}{1-3}$$

$$S_{10} = 442860$$

The answer is **C**.

NR2. Determine the first few terms of the series

100 + 112 + 125.44 + …

$a = 100$, $r = 1.12$, $n = 20$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$S_{10} = \frac{100\left(1-1.12^{20}\right)}{1-1.12}$$

$$S_{10} = 7205.24$$

The answer is **7205**

15.

$$\sum_{k=1}^{4} \log k = \log 1 + \log 2 + \log 3 + \log 4$$

$$= \log\left(1\times 2\times 3\times 4\right)$$

The answer is **C**.

NR 3. Write out the first few terms of the series:

4 + 16 + 64 + …

$a = 4$, $r = 4$, $S_n = 1\ 000\ 000$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$1000000 = \frac{4\left(1-4^n\right)}{1-4}$$

$$-3000000 = 4\left(1-4^n\right)$$

$$-750000 = 1-4^n$$

$$-750001 = -4^n$$

$$750001 = 4^n$$

$$\log 750001 = \log 4^n$$

$$\log 750001 = n\log 4$$

$$n = \frac{\log 750001}{\log 4}$$

$$n = 9.75 = 10$$

The answer is **10**.

16. Find a & r

$$a = -6, \quad r = \frac{9}{-6} = -\frac{3}{2}$$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$S_{11} = \frac{-6\left(1-(-1.5)^{11}\right)}{1-\left(-1.5\right)}$$

$$S_{11} = -210$$

The answer is **B**.

17. Write out the first few terms of the sequence

207, 190.44, 175.20, …

$t_n = 207\left(0.92\right)^{n-1}$

$t_8 = 207\left(0.92\right)^{8-1}$

$t_8 = 115.47$

The answer is **A**.

18.

$\sum_{k=3}^{13}\left(2^{k-1}\right) = 4+8+...4096$

The number of terms is (13 – 3) + 1 = 11

$S_n = \frac{a\left(1-r^n\right)}{1-r}$

$S_n = \frac{4\left(1-2^{11}\right)}{1-2}$

$S_{11} = 8188$

The answer is **D**.

19.

$a = 16$

$r = \frac{8\sqrt{2}}{16} = \frac{\sqrt{2}}{2}$

Write out the terms of the sequence

$16,\ 8\sqrt{2},\ 8,\ 4\sqrt{2}$

Side four = $4\sqrt{2}$

The answer is **B**.

20. Write out the first six terms representing side lengths

$16,\ 8\sqrt{2},\ 8,\ 4\sqrt{2},\ 4,\ 2\sqrt{2}$

Side six = $2\sqrt{2}$

Perimeter = $4\left(2\sqrt{2}\right) = 8\sqrt{2}$

The answer is **D**.

NR4. Write out the first few areas:

$A_1 = 16^2 = 256$

$A_2 = \left(8\sqrt{2}\right)^2 = 128$

$A_3 = 8^2 = 64$

The common ratio is $\frac{128}{256} = 0.5$

The answer is **0.5**.

21.
$a = 1$, $r = 2$, $n = 64$
Since we only want the amount on the 64th square, use the term formula

$$t_n = ar^{n-1}$$

$$t_{64} = 1(2)^{64-1}$$

$$t_{64} = 2^{63}$$

The answer is **C**.

22. Use the sum formula, since we have a limited number of loonies to fill all the squares.

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$700000000 = \frac{1(1-2^n)}{1-2}$$

$$-700000000 = 1-2^n$$

$$-700000001 = -2^n$$

$$700000001 = 2^n$$

$$\log 700000001 = n \log 2$$

$$n = 29.38 = 29$$

The answer is **B**.

23.

$$-2+\frac{4}{3}-\frac{8}{9}+\ldots$$

a = -2 $\quad r = \frac{4}{3} \div -2 = \frac{4}{3} \times -\frac{1}{2} = -\frac{4}{6} = -\frac{2}{3}$

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{-2}{1-\left(\frac{-2}{3}\right)}$$

$$S_\infty = -1.2$$

The answer is **C**.

24.
$a = 2$, $r = 0.7$, $n = 5$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{2(1-0.7^5)}{1-0.7}$$

$$S_n = 5.55$$

The answer is **D**.

25. $20+40+80+...+163840$

First determine the number of terms

$$t_n = ar^{n-1}$$
$$163840 = 20(2)^{n-1}$$
$$8192 = 2^{n-1}$$
$$2^{13} = 2^{n-1}$$
$$13 = n-1$$
$$n = 14$$

Now write out the sigma notation

$$\sum_{n=1}^{14} 20(2)^{n-1}$$

The answer is **B**.

NR5. $a = 15$, $r = 0.6$, $n = 9$

$$S_n = \frac{a(1-r^n)}{1-r}$$
$$S_9 = \frac{15(1-0.6^9)}{1-0.6}$$
$$S_9 = 37.1$$

Remember: this is not the final answer!

Multiply by 2 since each height happens twice due to bouncing, then subtract 15 because the initial fall only happens once.

Answer = **59.2**

26. For answer B, $r = x^2$,
for answer C, $r = -3$,
for answer D, $r = \sqrt{3}$

A is not geometric,
as there is no common ratio.

The answer is **A**.

27. If the sequence is geometric, the common ratio must be the same for all terms. Therefore,

$$\frac{2a-1}{3a} = \frac{7a+8}{2a-1}$$
$$(2a-1)(2a-1) = 3a(7a+8)$$
$$4a^2-4a+1 = 21a^2+24a$$
$$0 = 17a^2+28a-1$$

Solve for the x-intercepts in your calculator by graphing.

$a = -1.682$

It follows that the first term is
$3a = 3(-1.682) = -5.05$

The answer is **A**.

28. $-1, \frac{2}{3}, -\frac{4}{9}...$

$$a = -1,\ r = -\frac{2}{3}$$
$$t_{10} = -1\left(-\frac{2}{3}\right)^{10-1}$$
$$t_{10} = 0.026$$

The answer is **B**.

29.

$$\sum_{k=3}^{5} a+k = (a+3)+(a+4)+(a+5)$$
$$= 3a+12$$

The answer is **D**.

Written Response 1:

	Purchase Year					
	2006	**2005**	**2004**	**2003**	**2002**	**2001**
Current value as of Jan 1, 2006	\$800	\$840	\$882	**926.10**	**972.41**	**1021.03**

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$
$$S_6 = \frac{800\left(1-1.05^6\right)}{1-1.05}$$
$$S_6 = 5441.53$$

From table:

800 + 840 + 882 + 926.10 + 972.41 + 1021.03 = **5441.54**

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$
$$9500 = \frac{800\left(1-1.05^n\right)}{1-1.05}$$
$$9500 = \frac{800\left(1-1.05^n\right)}{-0.05}$$
$$-475 = 800\left(1-1.05^n\right)$$
$$-0.59375 = 1-1.05^n$$
$$-1.59375 = -1.05^n$$
$$1.59375 = 1.05^n$$
$$\log 1.59375 = \log 1.05^n$$
$$\log 1.59375 = n \log 1.05$$
$$n = 9.55$$

It will take a minimum of 10 years to have the required funds.

To solve the equation graphically, use the following:

$y_1 = 1.59375$
$y_2 = 1.05^x$

Use x: [-2, 10, 1] ; y: [-3, 3, 1]

Find the point of intersection; the x-value gives the solution.

*You could alternatively use
$y_1 = 9500$
$$y_2 = \frac{800\left(1-1.05^n\right)}{1-1.05}$$

Use x: [-2, 10, 1] ; y: [-100, 10000, 1]

Written Response 2:

- $-1+\frac{1}{2}-\frac{1}{4}+\ldots$

$$S_\infty=\frac{a}{1-r}$$

$$S_\infty=\frac{-1}{1-(-0.5)}$$

$$S_\infty=\frac{-1}{1.5}$$

$$S_\infty=\frac{-2}{3}$$

- $\sum_{k=1}^{\infty}100(0.3)^{k-1}$

$$S_\infty=\frac{a}{1-r}$$

$$S_\infty=\frac{100}{1-0.3}$$

$$S_\infty=142.86$$

-

$$S_\infty=\frac{a}{1-r}$$

$$\frac{13}{5}=\frac{a}{1-(-0.25)}$$

$$\frac{13}{5}=\frac{a}{1.25}$$

$$16.25=5a$$

$$a=\frac{13}{4}$$

-

$$S_n=\frac{a(1-r^n)}{1-r}$$

$$S_4=\frac{15(1-0.6^4)}{1-0.6}$$

$$S_4=32.64\,m$$

Multiply by 2 → 65.28
Subtract initial height → 50.28

-

$$S_\infty=\frac{a}{1-r}$$

$$S_\infty=\frac{15}{1-0.6}$$

$$S_\infty=37.5\text{ m}$$