

Math 12: Functions Review

1) Describe the transformation(s) that will occur if $y = f(x)$ is transformed as shown.

- a) $y = -f(5x)$ b) $y = 2f(x-3)$ c) $y = f(2(x+3))$ d) $2y = f(x)+6$
- e) $y = f^{-1}(x)$ f) $y = \frac{3}{2}f(-x)$ g) $y = f(\frac{3}{4}x-7)$ h) $y+4 = f(2-x)$

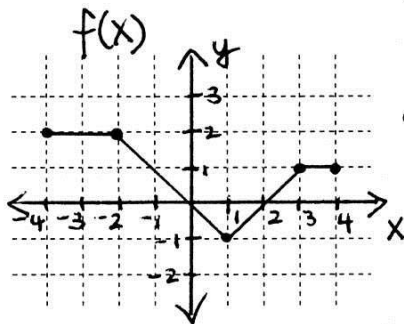
2) Write the equation for the function $y = f(x)$ after it has been transformed in the following manner.

- | | | |
|---|--|---|
| a) Vertical Expansion of 3
Vertical Translation up 6
Reflection over the y-axis
Horizontal Compression of 2/3
Horizontal Translation left 4 | b) Horizontal Expansion of 8
Horizontal Translation 9 right
Vertical translation down 3
Vertical Compression of 6/7
Reflection over the x-axis | c) Horizontal Translation 3 left
Horizontal Expansion of 7
Vertical Expansion of 5
Vertical Translation up 4
Reflection in the line $y=x$ |
|---|--|---|

3) Write the equation for the function $y = f(x)$ after it has been transformed in the following manner. Graph both the original and the transformed function.

- | | | |
|--|---|--|
| a) $y = \sqrt{4-(x+1)^2}-2$
Vertical Expansion of 2
Reflection over the y-axis
Horizontal Translation right 1 | b) $y = -\sqrt{(2x+3)}$
Horizontal Expansion of 4
Vertical Expansion 2
Vertical translation up 4 | c) $y = 2(x+1)^2$
Horizontal Translation 3 left
Vertical Compression of 1/4
Vertical Translation down 1 |
|--|---|--|

4) Given $y = f(x)$ graph the following



- | | | |
|-------------------|---|---|
| a) $y = f(x+3)+2$ | b) $y = -f(2x+3)-5$ | c) $y = \frac{1}{f(x)}$ |
| d) $y = 2f(-x+1)$ | e) $y = \frac{1}{2}f(\frac{1}{2}(x+1))$ | f) $y = f(x+1)-2 +3$ |

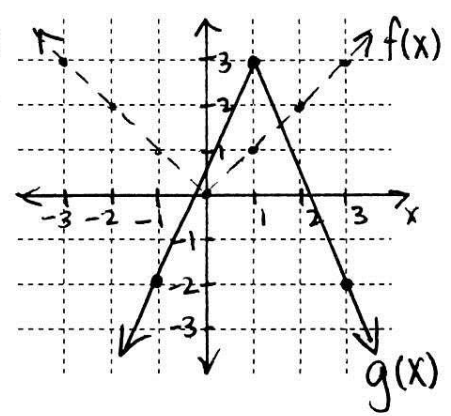
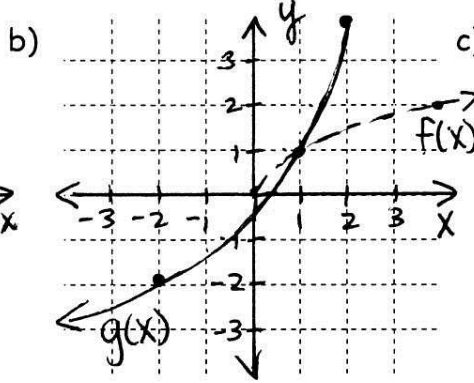
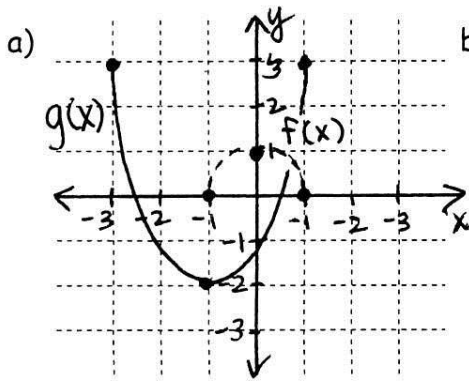
5) Write the inverses of the following

a) $y = \frac{3x-5}{5-4x}$

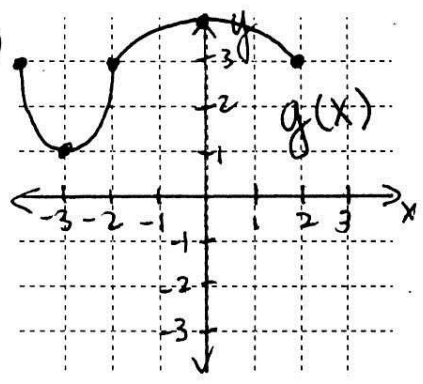
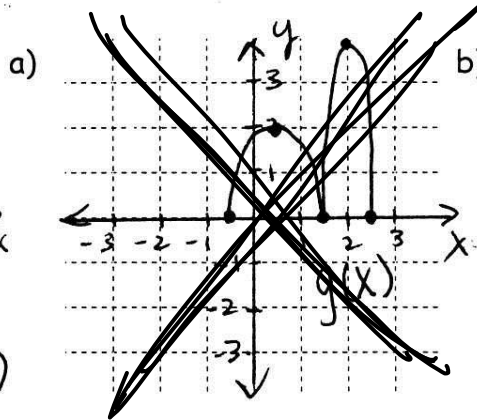
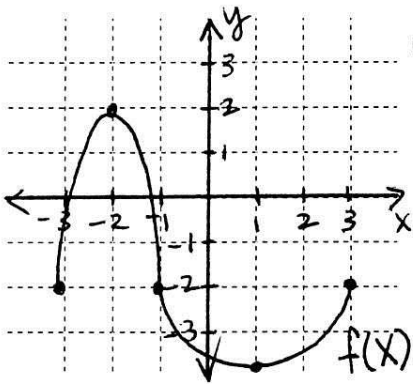
b) $y = \frac{2x}{4x+5}$

c) $y = 3x^2 - 6x + 7$

6) Determine the equation of $g(x)$ given the graph of $f(x)$



7) Given $f(x)$ determine the equation for $g(x)$



8) Determine the new equation for the function $f(x) = 2x^4 - 3x^3 - 4x^2 + 5x - 6$ if it is reflected in the following manner.

- a) Over the x-axis
- b) Over the y-axis
- c) Across the line $y=x$

9) The function $f(x) = k(x-a)^2 + b$ where $k < 0$, is translated 5 units down. What is its range?

Math 12: Functions Review Solutions

- 1) a) $y = f(5x)$ vertical compress by 1/5
- b) $y = 2f(x-3)$ vertical expand by 2, horizontal right 3
- c) $y = f(2(x+3))$ vertical compress by 1/2, horizontal left 3
- d) $2y = f(x) + 6$ vertical compress by 1/2, horizontal right 3

- e) $y = f^{-1}(x)$ inverse (reflection over the line $y=x$)
- f) $y = \frac{3}{2}f(-x)$ expand by 3/2, reflect over y-axis
- g) $y = f(\frac{3}{4}(x-7))$ vertical compress by 4/3, horizontal right 7
- h) $y + 4 = f(2-x)$ down 4, reflect over y-axis
- i) $y = 3f(x)$ vertical expand by 3
- j) $y = 3f(-x) + 6$ vertical expand by 3, reflect over y-axis
- k) $y = 3f(-\frac{3}{2}x) + 6$ vertical expand by 3, reflect over y-axis, horizontal compress by 2/3
- l) $y = 3f(\frac{3}{2}(x+4)) + 6$ vertical expand by 3, horizontal compress by 2/3, horizontal right 4

c) $y = f(x+3)$
 $y = f(1/7x+3)$
 $y = 5f(1/7x+3)$
 $y = 5f(1/7x+3)+4$

in all of these you start with a root function even before the "original" function!

3a) $y = \sqrt{4-(x+1)^2} - 2$
 semicircle $r=2$ left 1 & down 2

Original: $\begin{matrix} x & -3 & -1 \\ y & 2 & 0 \end{matrix}$

b) $y = -\sqrt{2x+3}$
 left 3 and compress 1/2 reflect over x

Original: $\begin{matrix} x & -1.5 & -1.5 \\ y & 0 & -1 \end{matrix}$

c) $y = 2(x+1)^2$
 VE of 2 left 1

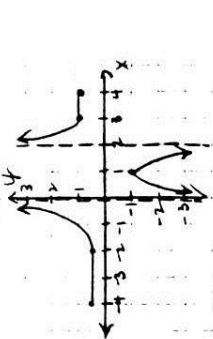
Original: $\begin{matrix} x & -2 & -1 & 0 \\ y & 2 & 0 & 2 \end{matrix}$

4a) $y = f(x+3) + 2$

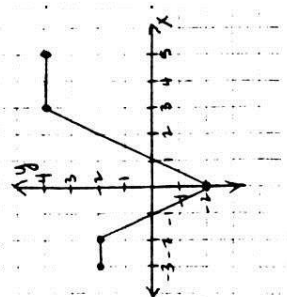
x	-4	-2	0	2	4
y	2	2	1	1	3

b) $y = -f(2x+3) - 5$

x	-1	1	3	5
y	-7	-7	-4	-4



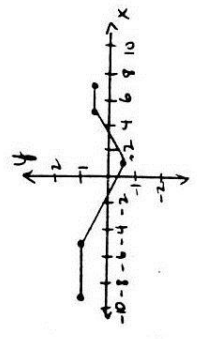
c) $y = \frac{1}{f(x)}$



d) $y = 2f(-x+1)$

x	1	2	3	4
y	5	4	3	2
	0	-2	2	2
	-3	2	2	

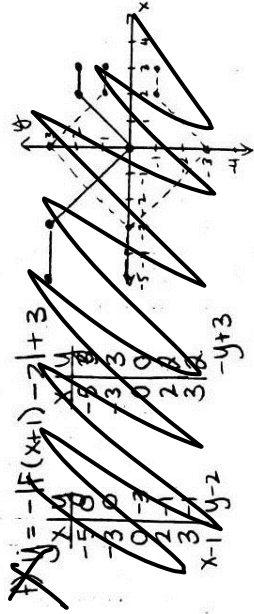
$-(x-1) \quad 2y$



e) $y = \frac{1}{2}f(\frac{1}{2}(x+1))$

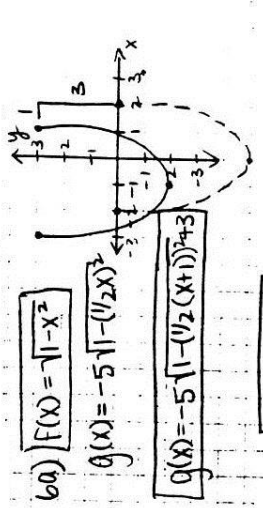
x	1	2	3	4
y	-9	-5	1	5
	1	-1/2	1/2	1/2

$2x-1 \quad 1/2y$

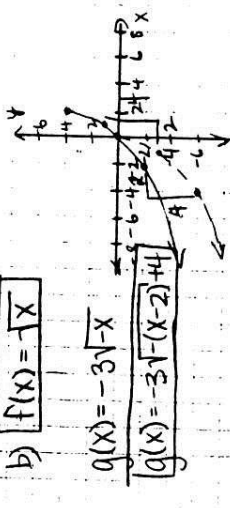


5) a) $x = \frac{3y-5}{5-4y}$
 $5x - 4xy = 3y - 5$
 $-4xy - 3y = -5x - 5$
 $y(4x - 3) = -5x - 5$
 $y = \frac{-5x-5}{4x-3}$

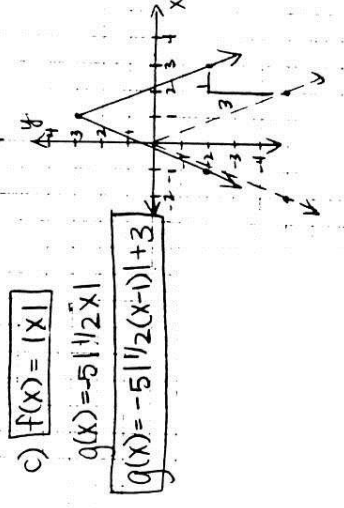
c) $y = 3x^2 - 6x + 7$
 $x = 3y^2 - 6y + 7$
 $x = 3(y^2 - 2y + 1) - 3 + 7$
 $x - 4 = (y-1)^2$
 $y - 1 = \pm \sqrt{x-3} + 1$



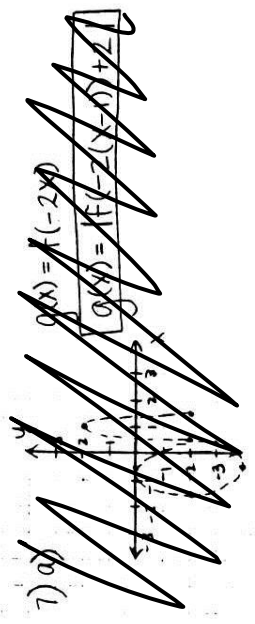
6a) $f(x) = \sqrt{1-x^2}$
 $g(x) = -5\sqrt{1-(1/2x)^2}$
 $g(x) = -5\sqrt{1-(1/2)(x+1)^2} + 3$



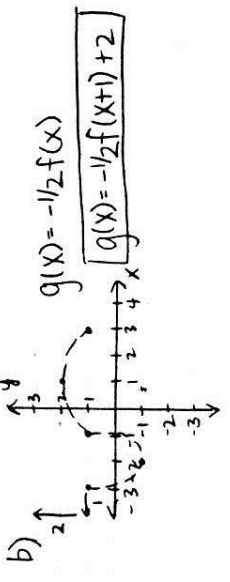
b) $f(x) = \sqrt{x}$
 $g(x) = -3\sqrt{x}$
 $g(x) = -3\sqrt{(x-2)+4}$



c) $f(x) = |x|$
 $g(x) = -5|1/2x|$
 $g(x) = -5|1/2(x-1)| + 3$



7) a) $g(x) = f(-2x)$
 $g(x) = f(-2(x-1)) + 2$



b) $g(x) = -1/2f(x)$
 $g(x) = -1/2f(x+1) + 2$

8) a) over $x \Rightarrow -f(x)$
 so, $y = -2x^4 + 3x^3 + 4x^2 - 5x + 6$

b) over $y \Rightarrow f(-x)$
 so, $y = 2x^4 + 3x^3 - 4x^2 - 5x - 6$

c) across $y = x$
 so, $x = 2y^4 - 3y^3 - 4y^2 + 5y - 6$

1) $(a,b) \Rightarrow 5$ down $(a,b-5)$
 $k < 0$
 $R: y \leq b-5$